
Membership-based Synthesis of Linear Hybrid Automata

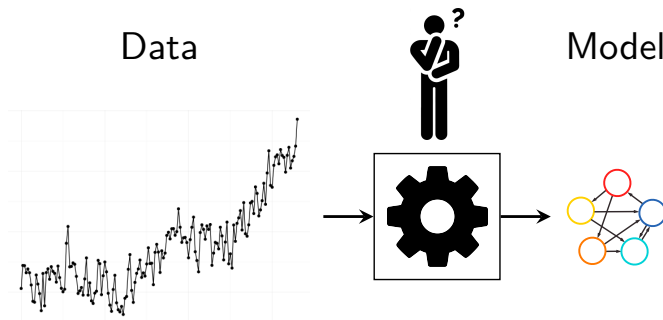
Christian Schilling

July 16, 2019, CAV

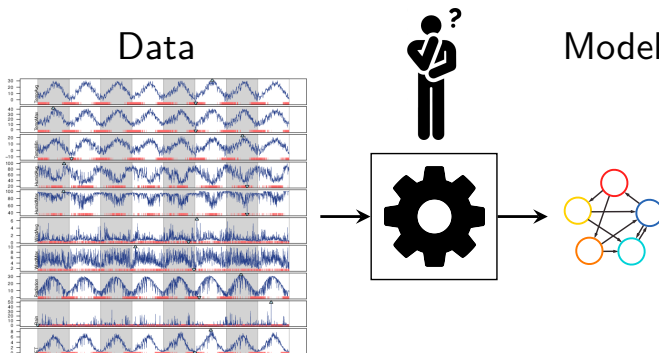


joint work with
Miriam García Soto, Thomas A. Henzinger, Luka Zeleznik

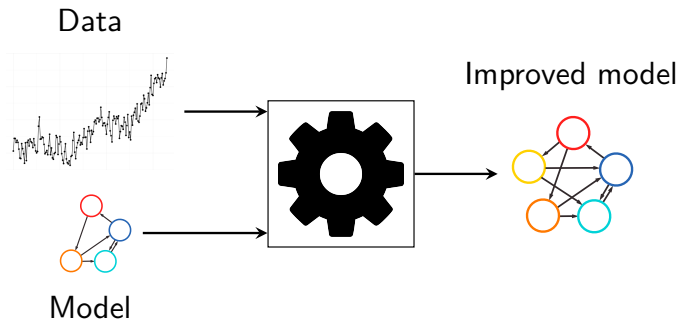
Model synthesis from data



Model synthesis from data



Adaptive synthesis algorithm



Overview

Introduction

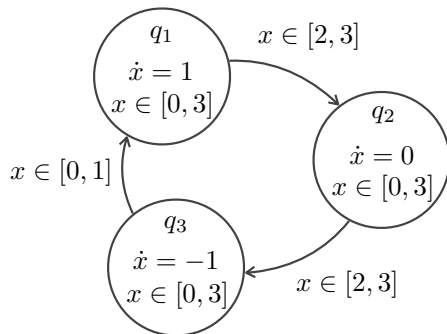
Preliminaries

Synthesis

Evaluation

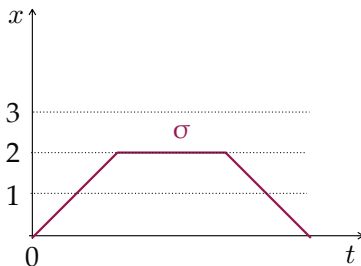
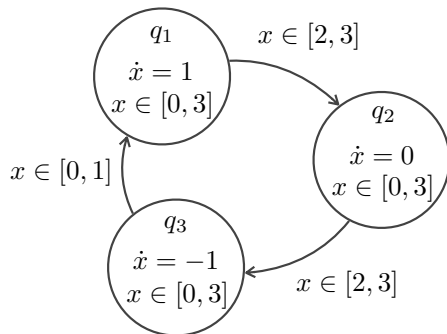
Summary

Linear hybrid automaton (LHA) model

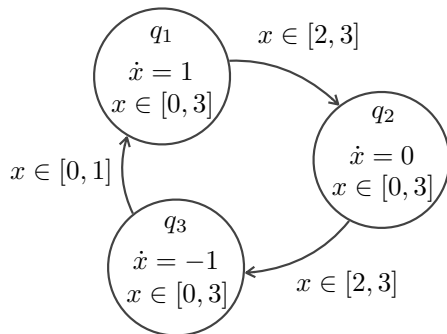


Linear hybrid automaton (LHA) model

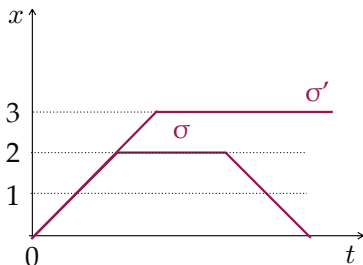
Execution



Linear hybrid automaton (LHA) model

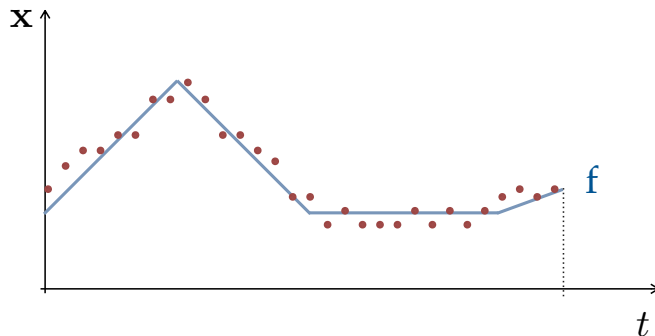


Executions

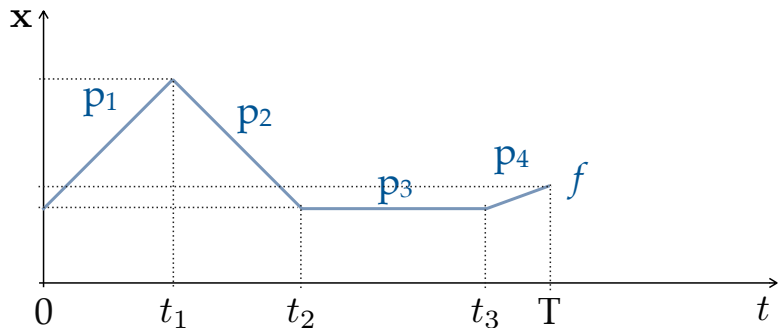


- Nondeterministic mode changes
- Restriction in this work: continuous executions

Piecewise-linear (PWL) function



Piecewise-linear (PWL) function



Related work

- SARX models (discrete time, deterministic switching) and PWARX models (SARX with state-space partition) can be synthesized algebraically¹; some adaptive algorithms exist^{2,3}
- Existing approaches for hybrid automata are not adaptive and come with limitations (e.g., periodic⁴, acyclic⁵, stateless⁶, deterministic⁷)

¹S. Paoletti et al. *Eur. J. Control* (2007).

²A. Skeppstedt et al. *Int. J. Control* (1992).

³Y. Hashambhoy and R. Vidal. *CDC*. 2005.

⁴R. Grosu et al. *HSCC*. 2007.

⁵O. Niggemann et al. *AAAI*. 2012.

⁶D. L. Ly and H. Lipson. *JMLR* (2012).

⁷I. Lamrani et al. *ICPS*. 2018.

Overview

Introduction

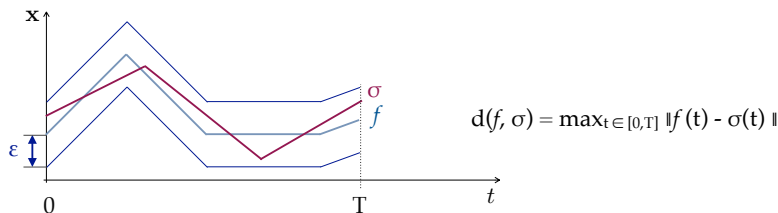
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Synthesis problem



ϵ -capturing

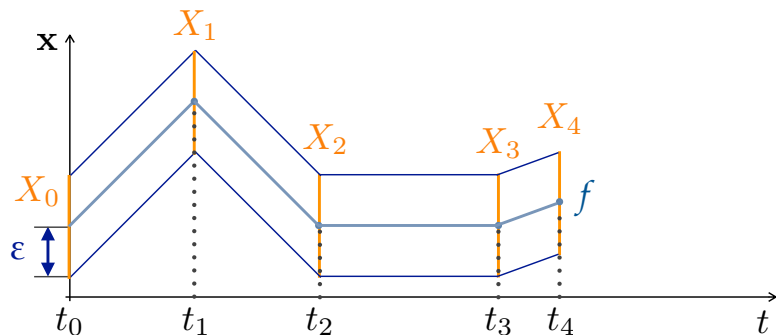
An LHA $\mathcal{H}$ ϵ -captures a PWL functions f if there exists an execution σ with $d(f, \sigma) \leq \epsilon$

Synthesis problem

Given a finite set of PWL functions $\mathcal{F}$ and $\epsilon \in \mathbb{R}_{\geq 0}$, construct an LHA $\mathcal{H}$ that ϵ -captures all $f \in \mathcal{F}$

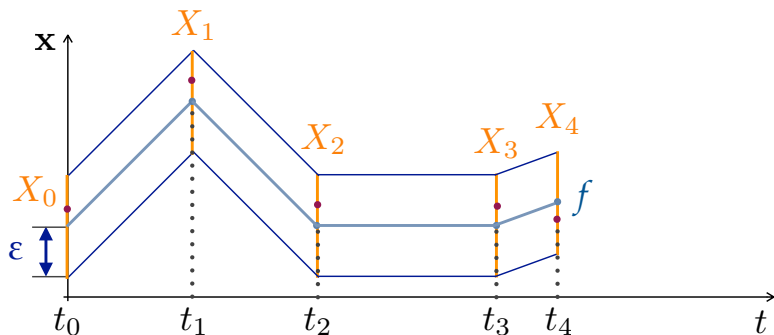
Synchronous switching

- Execution σ must switch synchronously with PWL function f



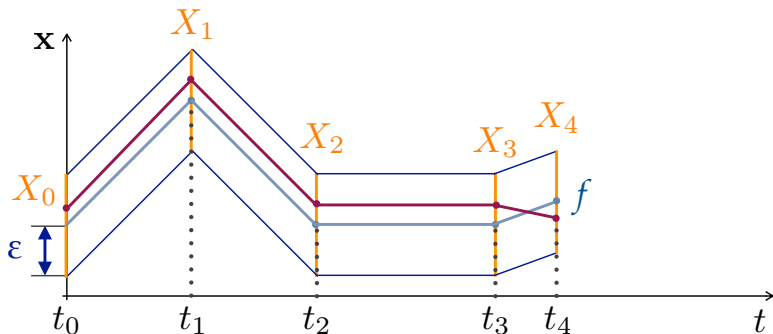
Synchronous switching

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Synchronous switching

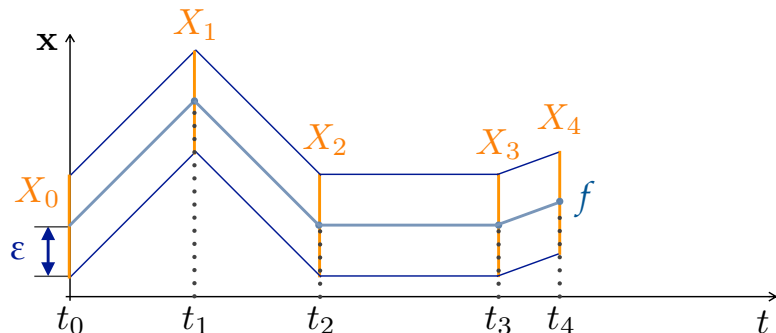
- Execution σ must switch synchronously with PWL function f



- Reduction to satisfiability of linear-arithmetic formula
 - Parametric in number of modes (i.e., can be minimized)

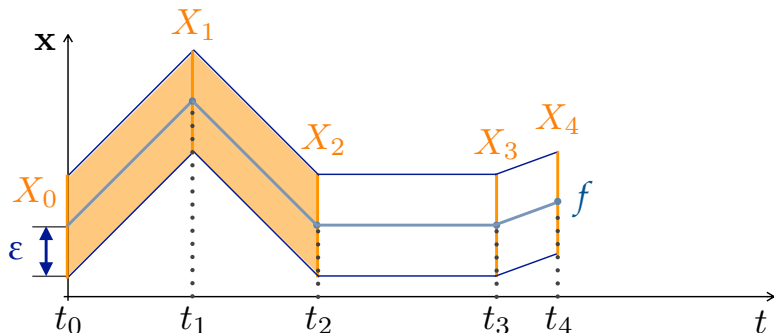
Asynchronous switching

- Execution σ must switch in intervals close to PWL function f



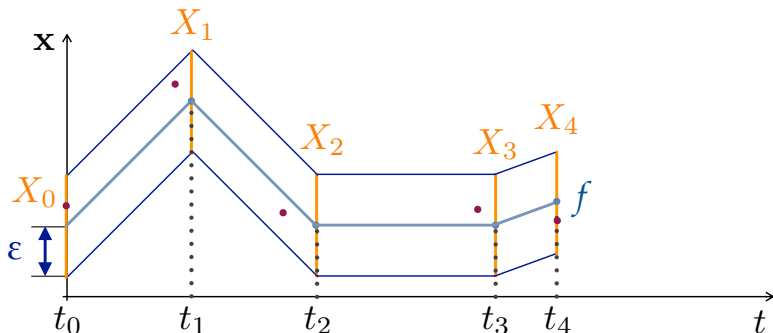
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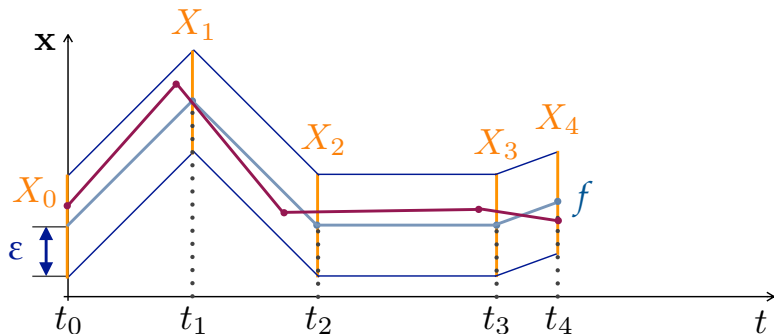
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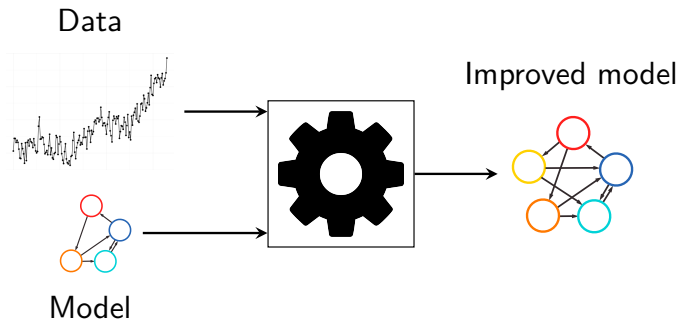
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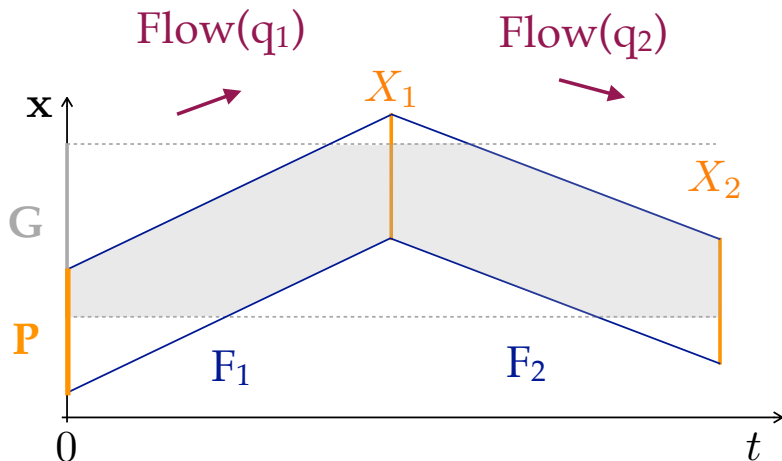


- Counterexample-guided, based on membership test

Recall: Adaptive synthesis algorithm

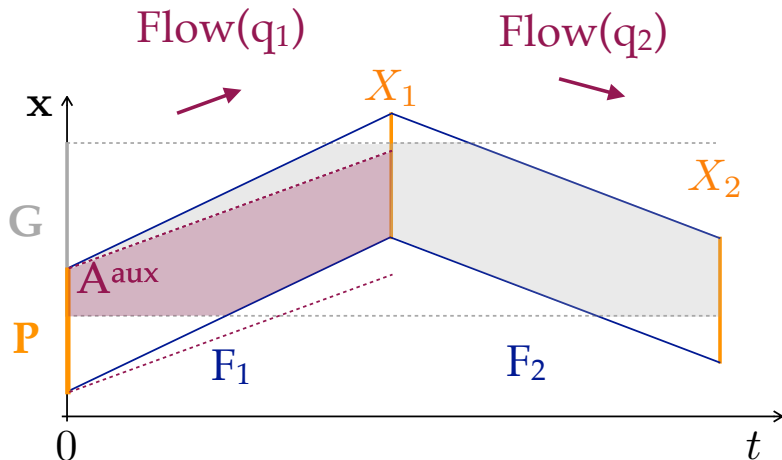


Membership algorithm

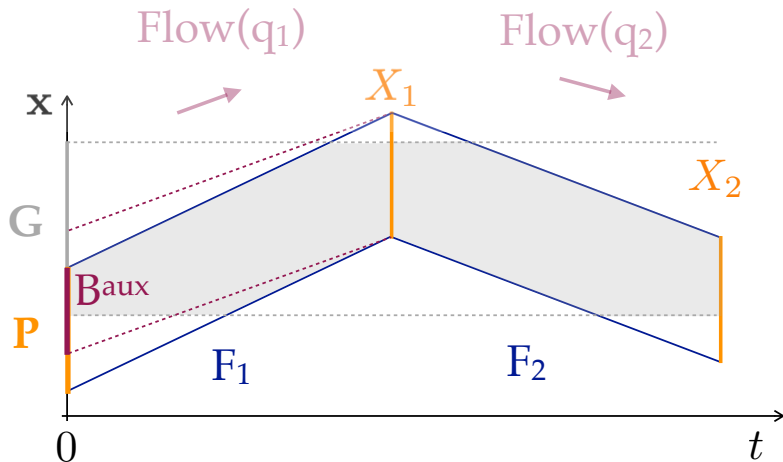


- Example: one step along path with prefix q_1q_2

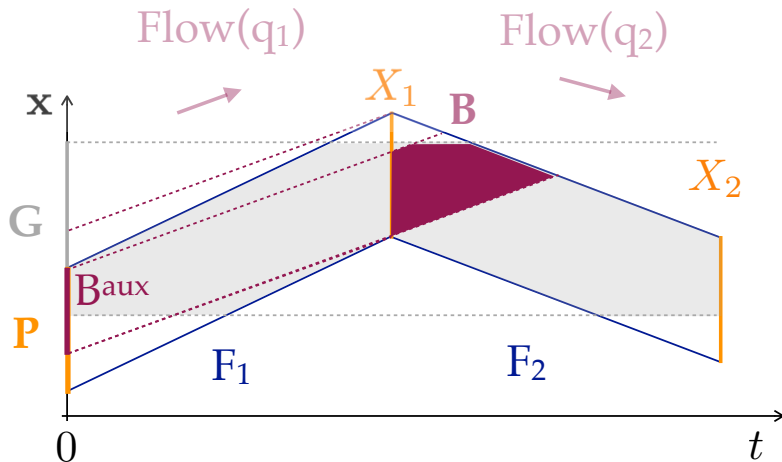
Membership algorithm



Membership algorithm

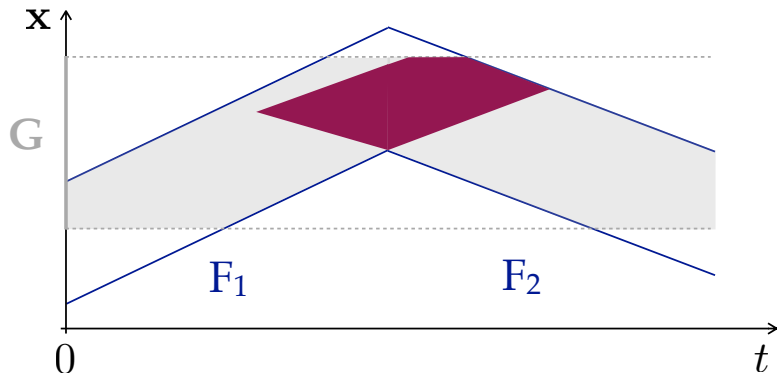


Membership algorithm



Membership algorithm

$$\mathbf{P} = \mathbf{A} \cup \mathbf{B}$$



- PWL function is ε -captured along path iff last set is nonempty

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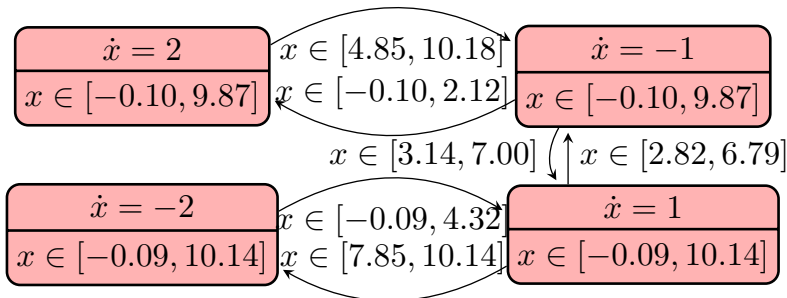
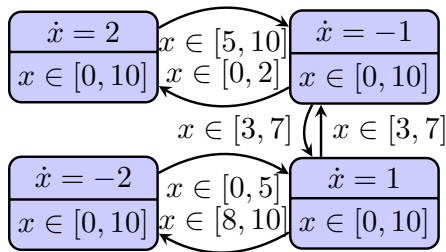
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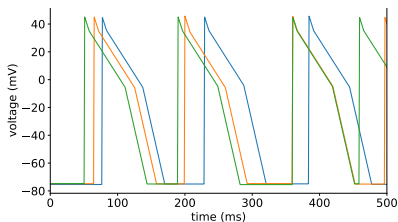
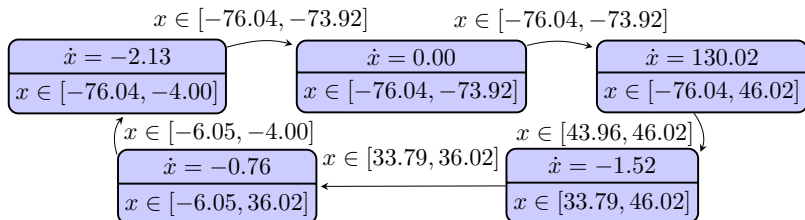
Summary

Synthetic model replication

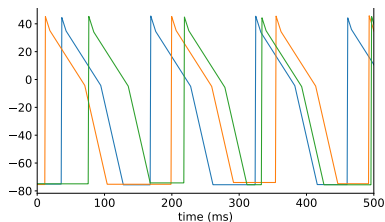
$$\varepsilon = 0.2$$



Voltage traces of excitable cell



Sample input traces



Sample executions

Summary

- Automatic synthesis of linear hybrid automaton from piecewise-linear functions
- Trade-off parameter ε (model size vs. model precision)
- Model with synchronous switching
 - Reduction to linear arithmetic
 - Minimal number of modes
- Model with asynchronous switching
 - Adaptive algorithm
 - Based on membership/reachability queries
 - Sound and complete for a general class of LHA