To safe AI control systems in three steps



INDEPENDENT RESEARCH

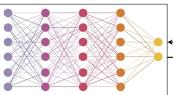
FUND DENMARK

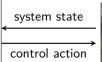
Christian Schilling













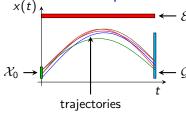
Neural-network controller

Environment

Reach-avoid specification and reach-avoid problem

Reach-avoid specification:

- Given:
 - Set of initial states $\mathcal{X}_0 \subseteq \mathbb{R}^n$
 - Set of **goal states** $\mathcal{G} \subseteq \mathbb{R}^n$
 - Set of error states $\mathcal{E} \subseteq \mathbb{R}^n$
 - Time bound T
- Aim: starting at \mathcal{X}_0 , reach \mathcal{G} within time T while avoiding \mathcal{E}
- Covers many real-world scientific and engineering problems



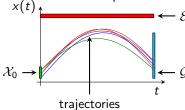
Reach-avoid specification and reach-avoid problem

Reach-avoid specification:

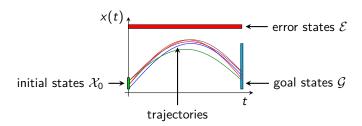
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Reach-avoid problem:

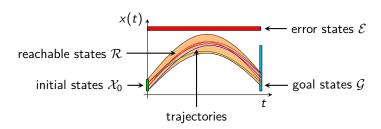
- Does a given system satisfy a reach-avoid specification?
- Undecidable for nonlinear dynamics



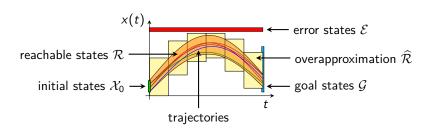
Verify: all trajectories lead to goal states avoiding error states



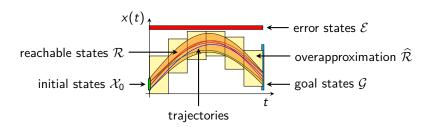
- Verify: all trajectories lead to goal states avoiding error states
- Equivalent to computing the reachable states $\mathcal R$ and proving $\mathcal R\cap\mathcal E=\emptyset$ \wedge $\mathcal R_{\mathcal T}\subseteq\mathcal G$
- ullet R only computable under strong restrictions



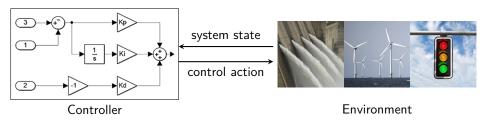
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- Challenge in practice: trade-off between precision and scalability



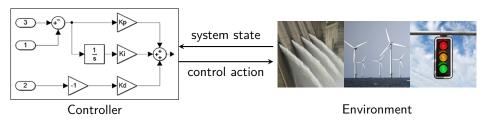
Research problems for control systems



Reach-avoid problem:

- Does a given controller satisfy a reach-avoid specification?
- Undecidable for nonlinear dynamics
- Need ways to compute the reachable states $\mathcal R$ (resp. an overapproximation $\widehat{\mathcal R}$) for both the controller and the environment

Research problems for control systems



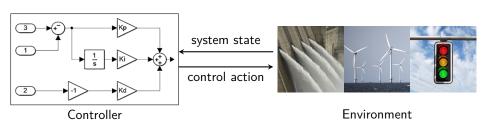
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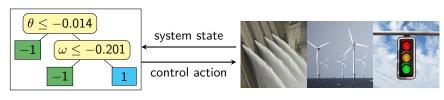
Synthesis problem:

- Find a controller that satisfies a reach-avoid specification
- Even harder problem

• Traditionally, the controller is designed by engineers



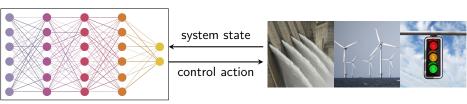
- Traditionally, the controller is designed by engineers
- Now we can machine-learn high performance controllers



Decision-tree controller

Environment

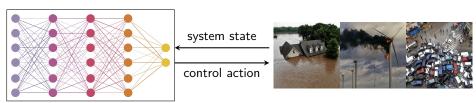
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Neural-network controller

Environment

- Traditionally, the controller is designed by engineers
- Now we can machine-learn high performance controllers
- No safety guarantee and often intransparent ("black box")

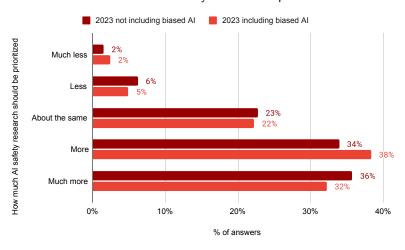


Neural-network controller

Environment

Al safety¹

"How much should AI safety research be prioritized?"



¹Grace et al. (2023). Thousands of Al authors on the future of Al.

Environment:

 $\dot{x} = v \cos(\theta)$ $\dot{y} = v \sin(\theta)$

 $\dot{v} = \mathbf{u_1} + \mathbf{w}$

 $\dot{\theta} = \mathbf{u_2}$



Environment:

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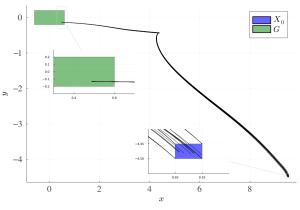
$$\dot{\theta} = \mathbf{u}_2$$

Specification:

$$x(0) \in \mathcal{X}_0$$

$$x(10) \stackrel{!}{\in} \mathcal{G}$$

Controller:



42 simulations

Environment:

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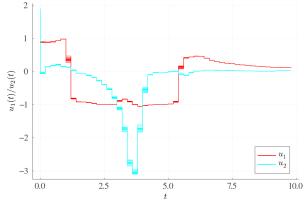
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control signals (42 simulations)

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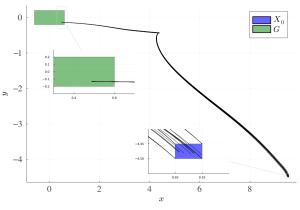
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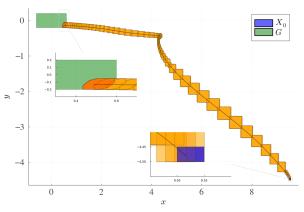
Al control

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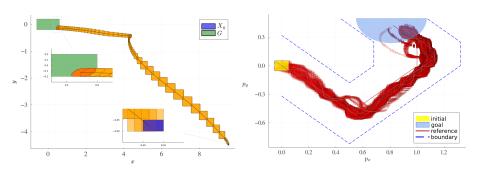


Overapproximated reachable states

To safe AI control systems in three steps

Step 1: Verification

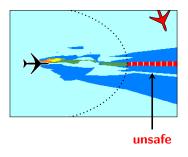
- Goal: Given a controller, show that it satisfies the specification
- Focus: Precise, scalable, and quantitative verification techniques



- S, Forets, and Guadalupe. AAAI. 2022. Kochdumper, S, Althoff, and Bak. NASA Formal Methods. 2023.
- S, Lukina, Demirović, and Larsen. NeurIPS. 2023.

Step 2: Falsification

- Goal: Given a controller, show that it violates the specification
- Focus: Targeted falsification techniques



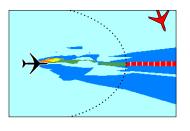
maximal cuter flowpipe
maximal inner flowpipe
robust duter flowpipe
robust inner flowpipe
minimal cuter flowpipe
minimal outer flowpipe
minimal inner flowpipe

Constrained optimization

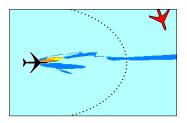
Underapproximated reach tube

Step 3: Repair / Synthesis

- Goal: Given a violating controller, make it satisfy the specification falsify → repair → verify
- Focus: Repair techniques for high controller performance

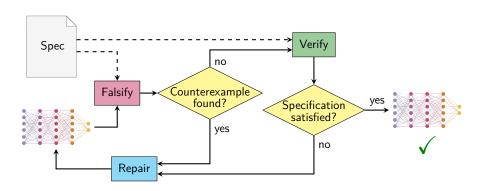




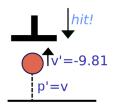


After repair

Repair loop guided by counterexamples



Learned controller guarded by synthesized shield



Vision

