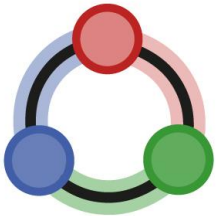
Reachability analysis and neural-network controlled systems



JuliaReach

Marcelo Forets

UNIVERSIDAD DE LA REPÚBLICA URUGUAY

CURE Centro Universitario Regional del Este

Christian Schilling

AALBORG UNIVERSITY DENMARK

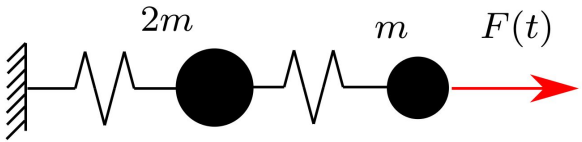
Presentation at

julialab Accelerating Science through Computing

Massachusetts Institute of Technology

19th Oct' 2021

What is reachability?



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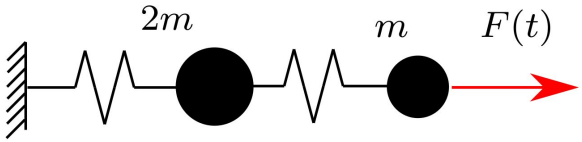

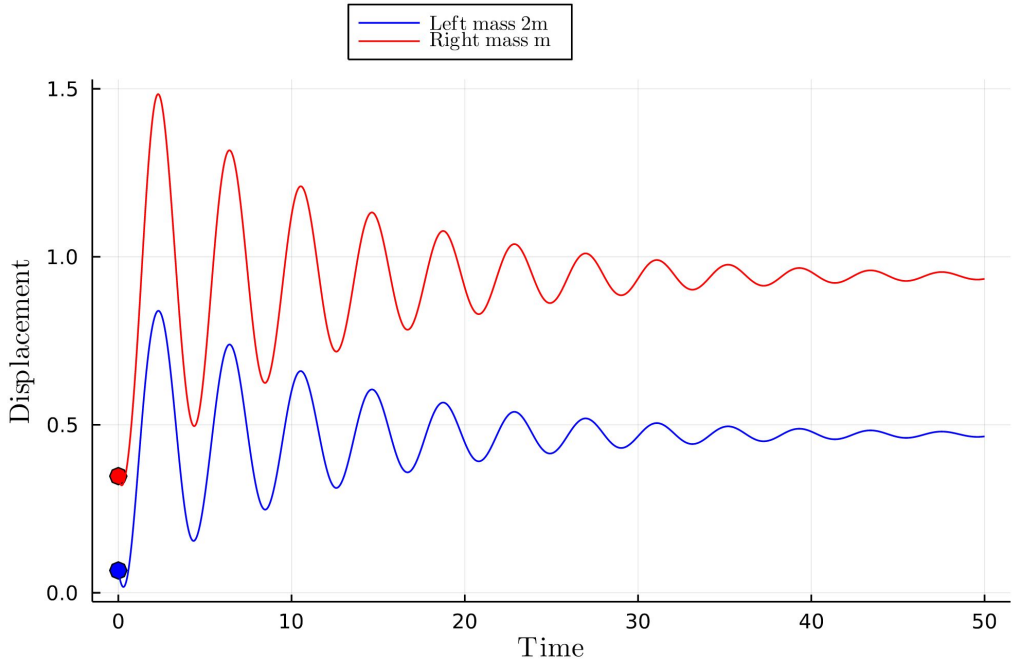

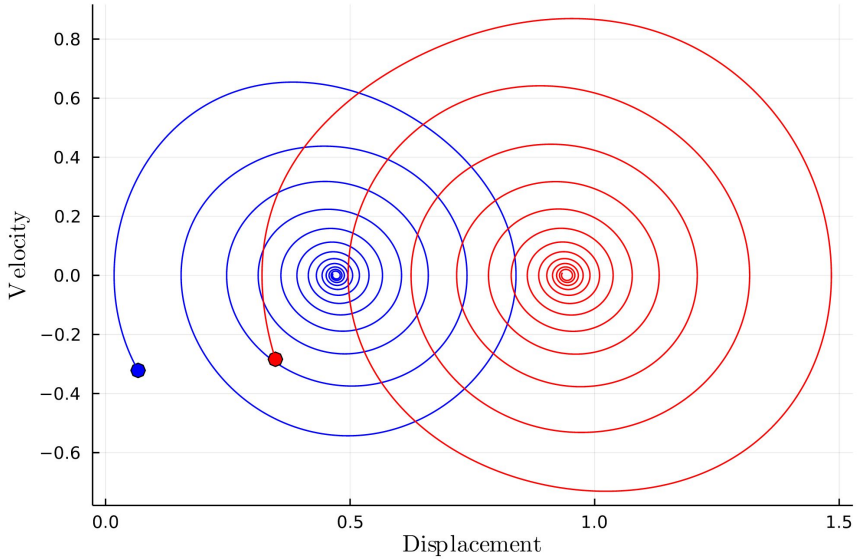

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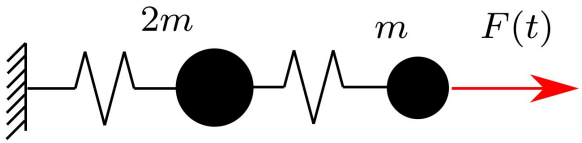

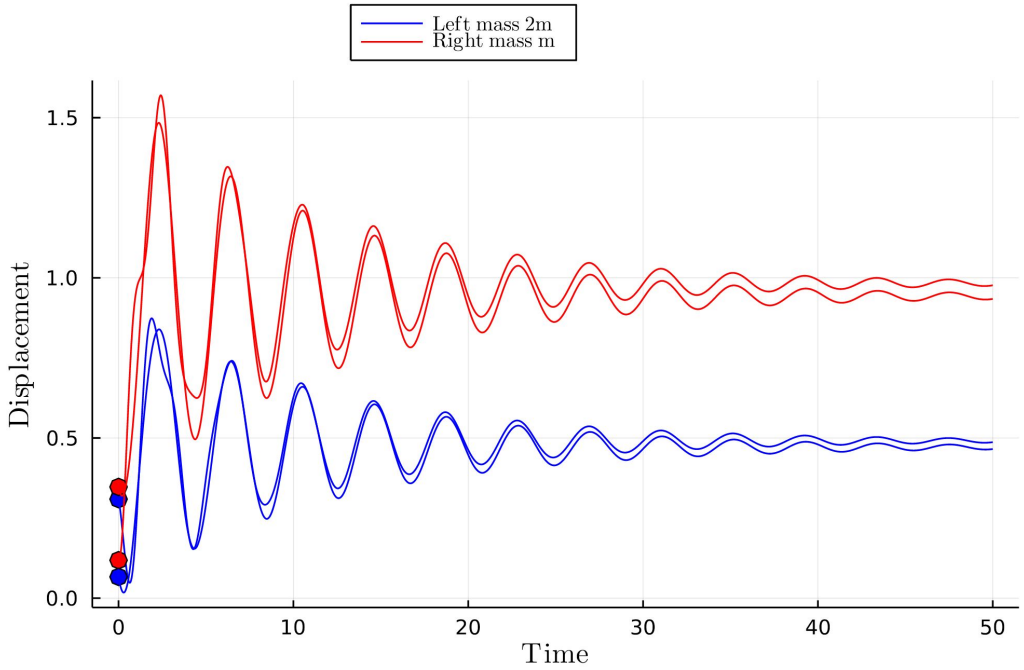

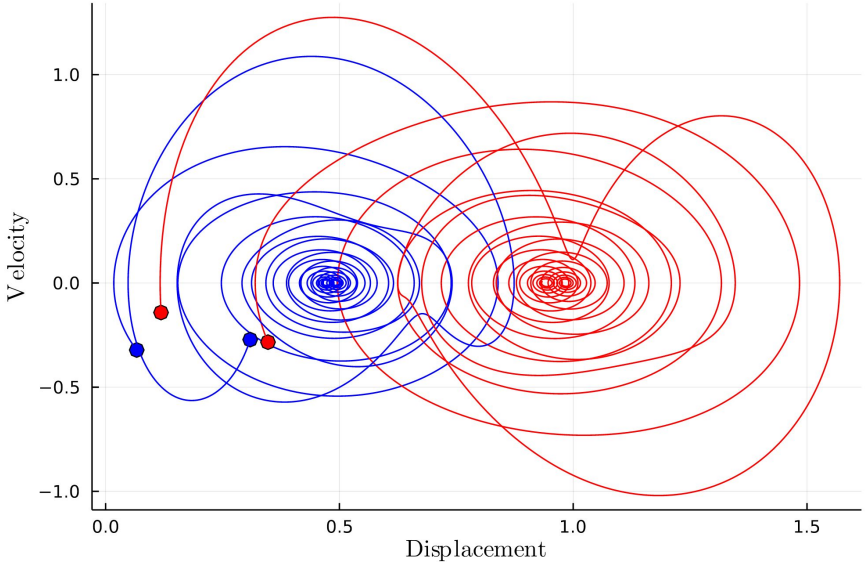

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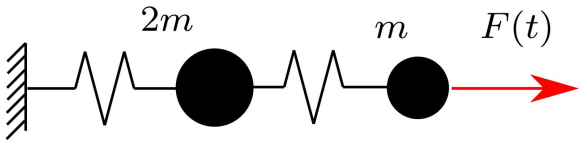

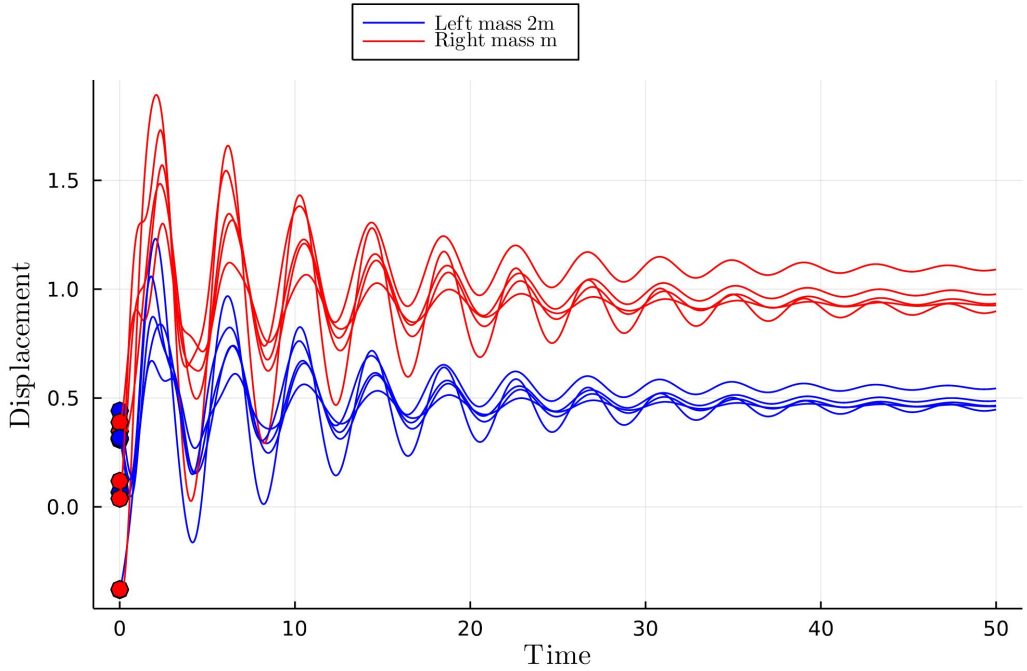

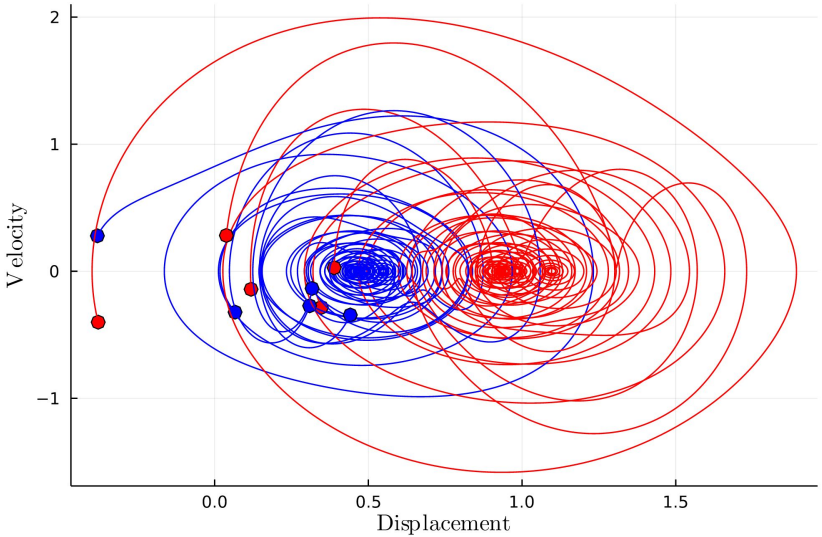

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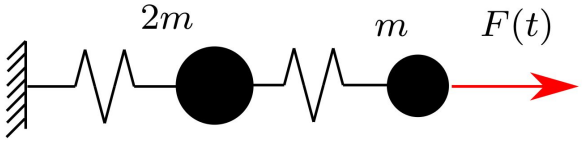

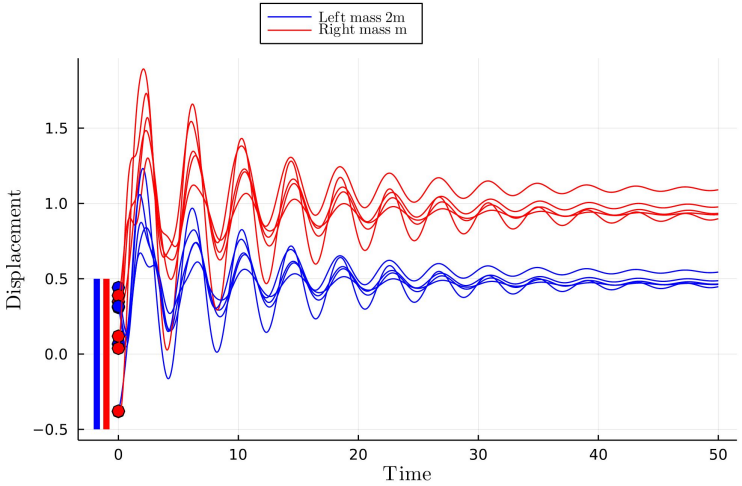

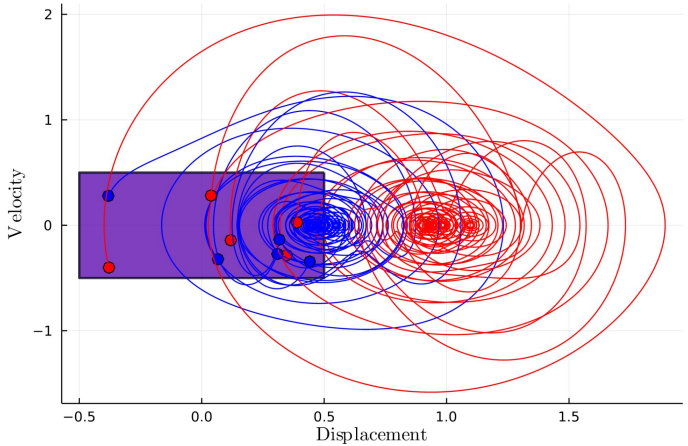

Reachability is a numerical method to compute sets of states reachable by dynamical systems for *all initial states* and all admissible *parameters* and *inputs*

Example taken from https://github.com/JuliaReach/SetPropagation-FEM-JuliaCon21, presented at the JuliaCon'2021 Set Propagation Methods in Julia: Techniques and Applications. **Submitted to JuliaCon'21 Proceedings (extended abstract).**

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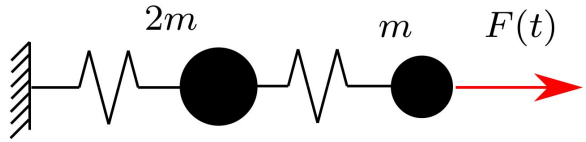

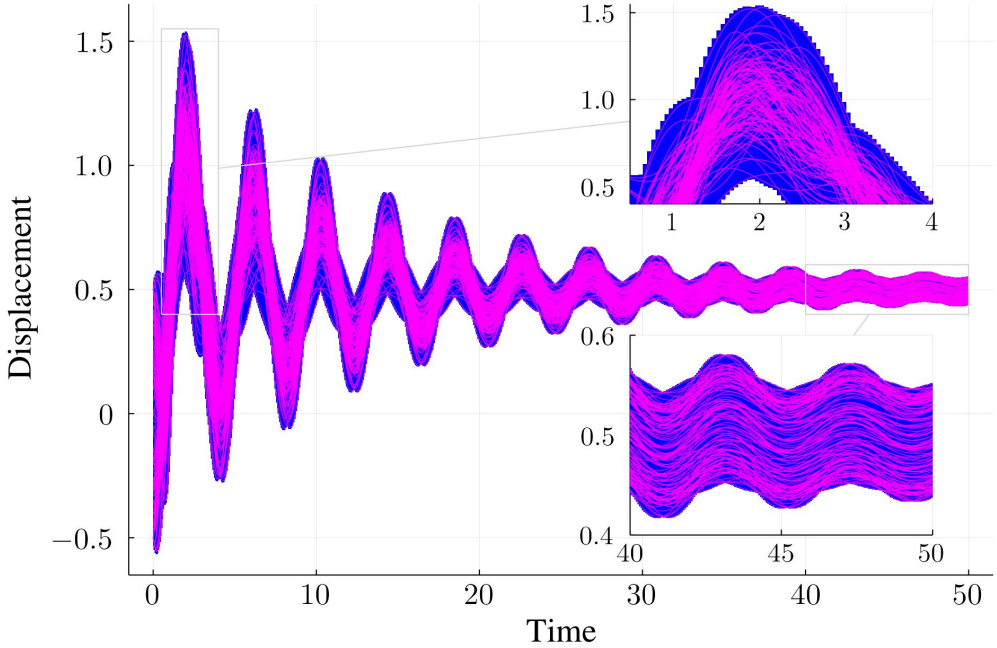


```julia
using ReachabilityAnalysis

# model parameters
m = 0.25; k = 2.0

# finite-element method assembled matrices
M = [2m 0; 0 m]; K = [2k -k; -k k]; C = (M+K)/20
F = [0.0, 1.0]; ΔF0 = Interval(0.9, 1.1)

# initial-value problem with uncertain initial conditions
U0 = BallInf(zeros(4), 0.5)
sys = SecondOrderLinearContinuousSystem(M, C, K, F)
prob = InitialValueProblem(homogeneize(sys), U0 × ΔF0)

# solve using support function method (box directions)
solA = solve(prob, 50, LGG09(δ=5e-2, dirs=:box, dim=5))
```

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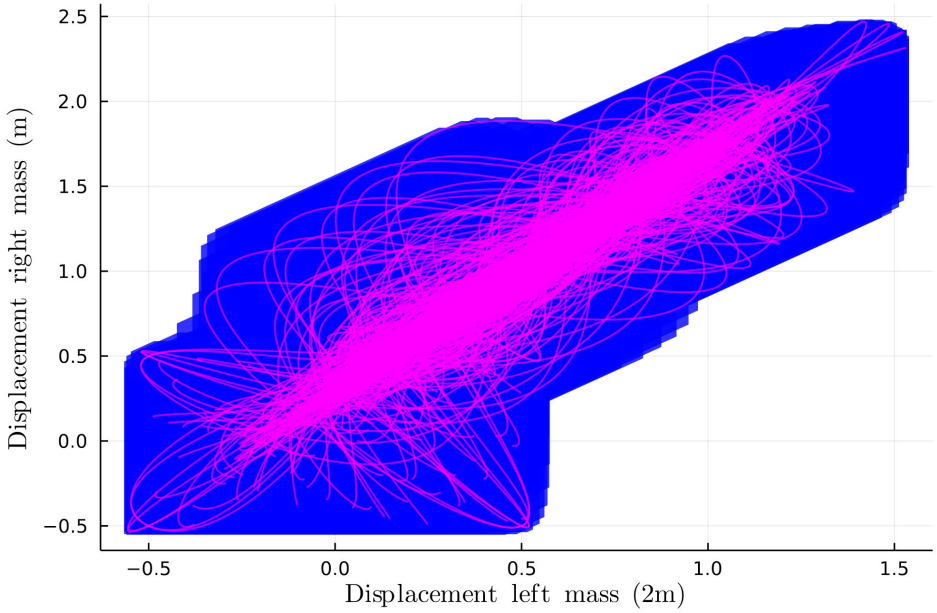
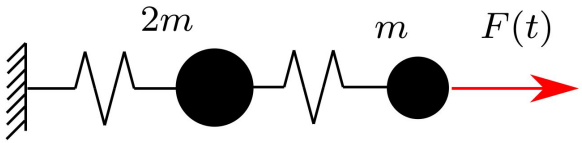
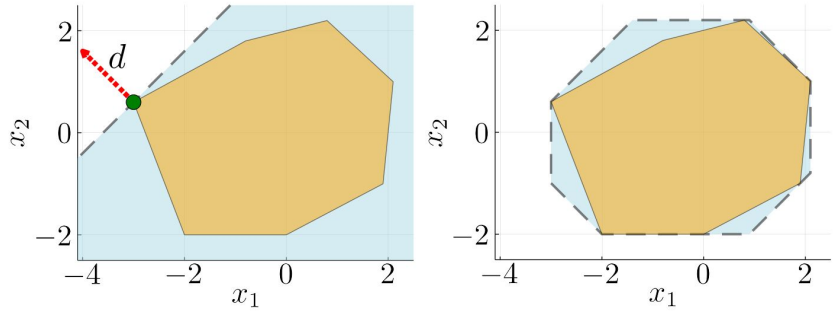





```
1 # solve using octagonal template directions
2 solB = solve(prob, 50, LGG09(δ=5e-2, dirs=:oct, dim=5))
```



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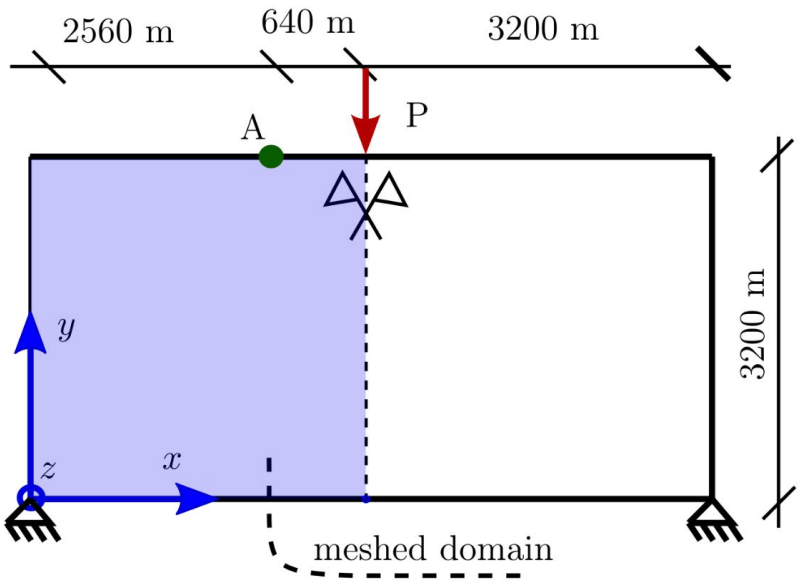
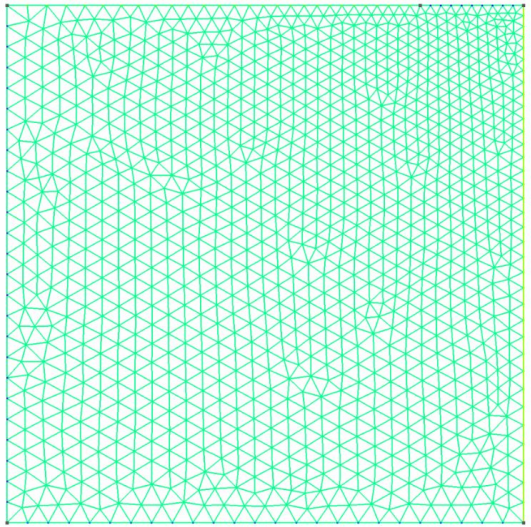


(a) Diagram of domain and boundary conditions considered.



(b) Finite Element Method mesh used, formed by triangular elements.

2235 linear triangular elements

1180 nodes

See: *Combining Set Propagation with Finite Element Methods for Time Integration in Transient Solid Mechanics Problems*. Forets, Marcelo, Daniel F. Caporale, and Jorge M. Pérez Zerpa. arXiv preprint arXiv:2105.05841.
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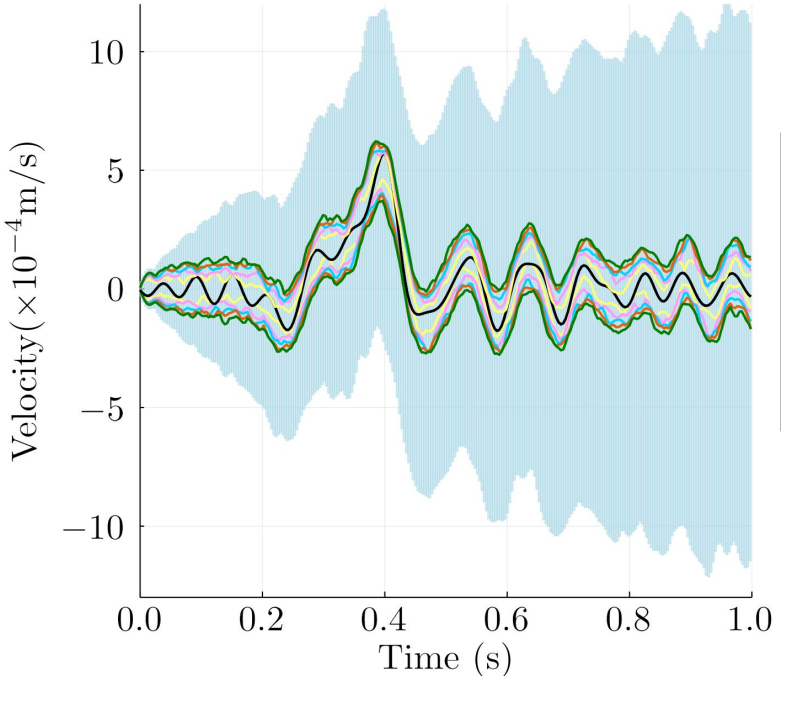

Method	# Trajectories	Time (s)	$\|v_{env}\|_{L_1}$ (10^{-5})	$\|v_{env}\|_{L_\infty}$ (10^{-5})
Newmark	1	0.3	9.27	56.98
Newmark	10	2.0	13.52	57.53
Newmark	100	17.7	16.61	57.59
Newmark	1000	175.5	18.52	58.22
Newmark	10000	1771.4	19.98	61.18
Newmark	100000	17796.1	21.42	62.21
Set Propagation	-	8.5	81.33	122.25

See: *Combining Set Propagation with Finite Element Methods for Time Integration in Transient Solid Mechanics Problems*.
Forets, Marcelo, Daniel F. Caporale, and Jorge M. Pérez Zerpa. arXiv preprint arXiv:2105.05841 (2021).
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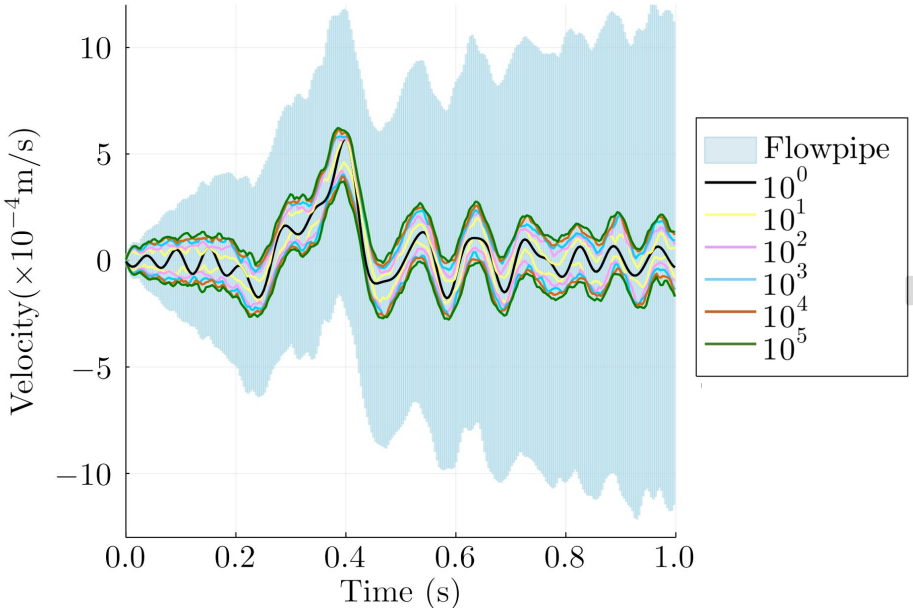

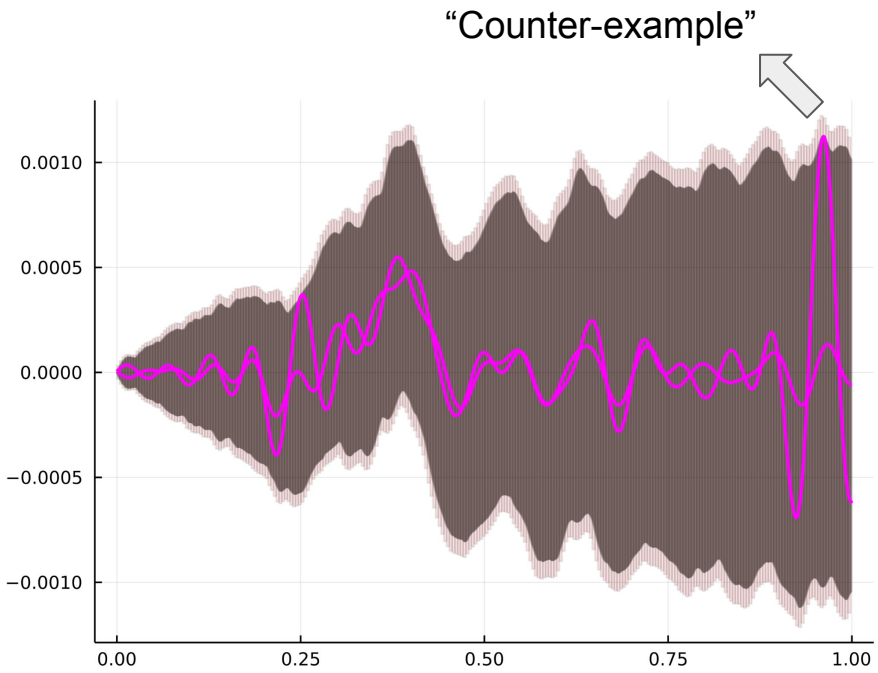

Reachability produces a sequence of sets (flowpipe) that converge to the exact reachable states when the time-step decreases

See: *Combining Set Propagation with Finite Element Methods for Time Integration in Transient Solid Mechanics Problems*. Forets, Marcelo, Daniel F. Caporale, and Jorge M. Pérez Zerpa. arXiv preprint arXiv:2105.05841 (2021).
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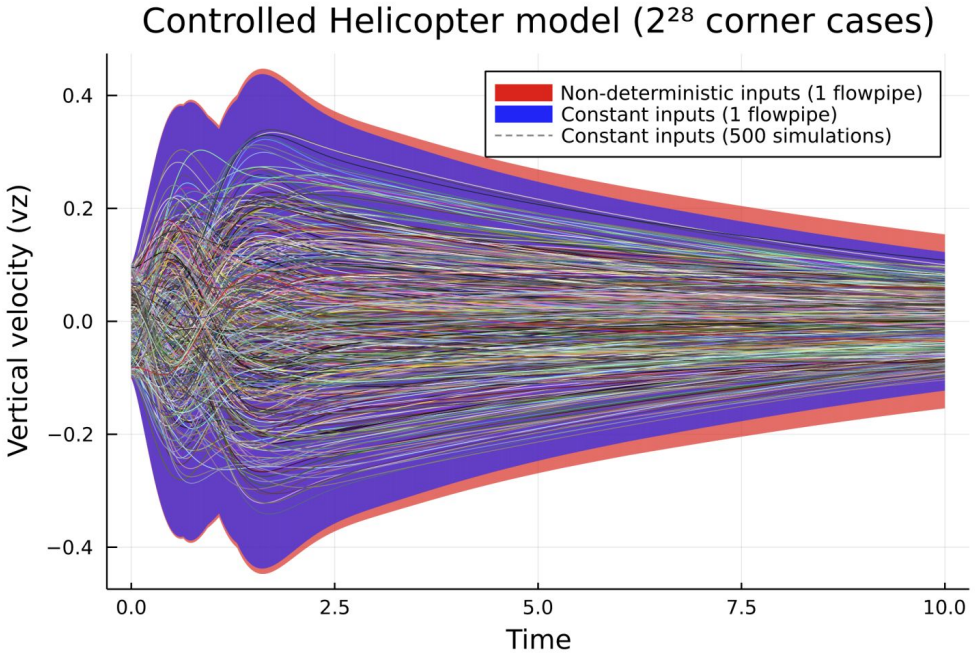
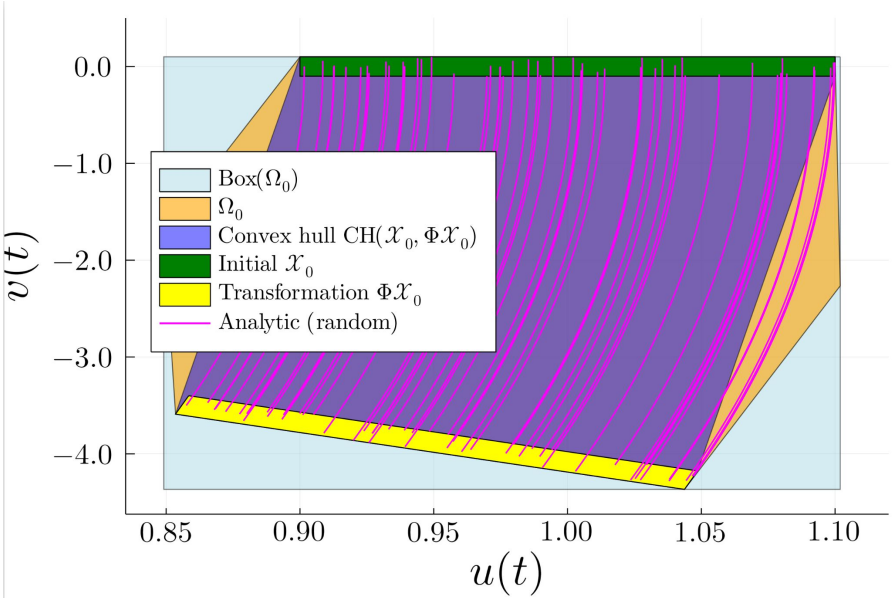


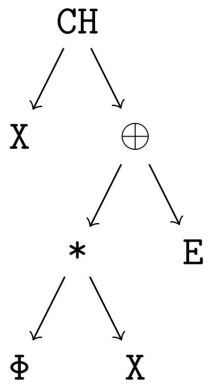
Reachability can be used to model *uncertain initial states*, non-deterministic *inputs*, non-deterministic *transitions* in hybrid systems, uncertain *parameters* and *noise*.

See: *Set propagation techniques for reachability analysis.* Matthias Althoff, Goran Frehse, and Antoine Girard. *Annual Review of Control, Robotics, and Autonomous Systems* 4 (2021): 369-395.

LazySets.jl: Scalable Symbolic-Numeric Set Computations



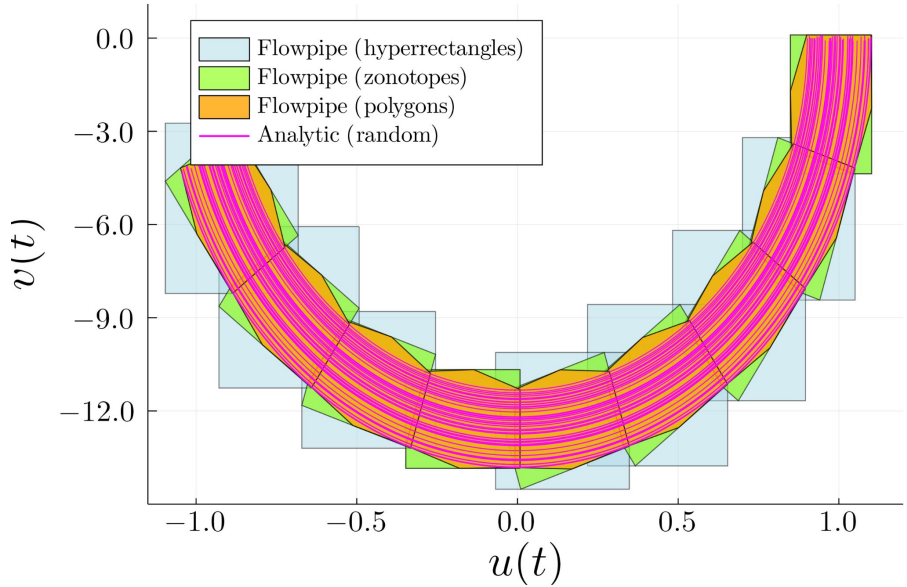

```
julia> Ω0 = CH(X0, Φ*X0 ⊕ E+)
```

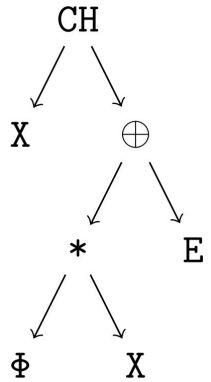


Efficient set computations require specialized algorithms based on different combinations of *set type representations* and *operations* involved.

See: *LazySets.jl: Scalable Symbolic-Numeric Set Computations.* Marcelo Forets and Christian Schilling. arXiv preprint arXiv:2110.01711 (2021). **Submitted to JuliaCon'2021 (full paper).**

LazySets.jl: Scalable Symbolic-Numeric Set Computations





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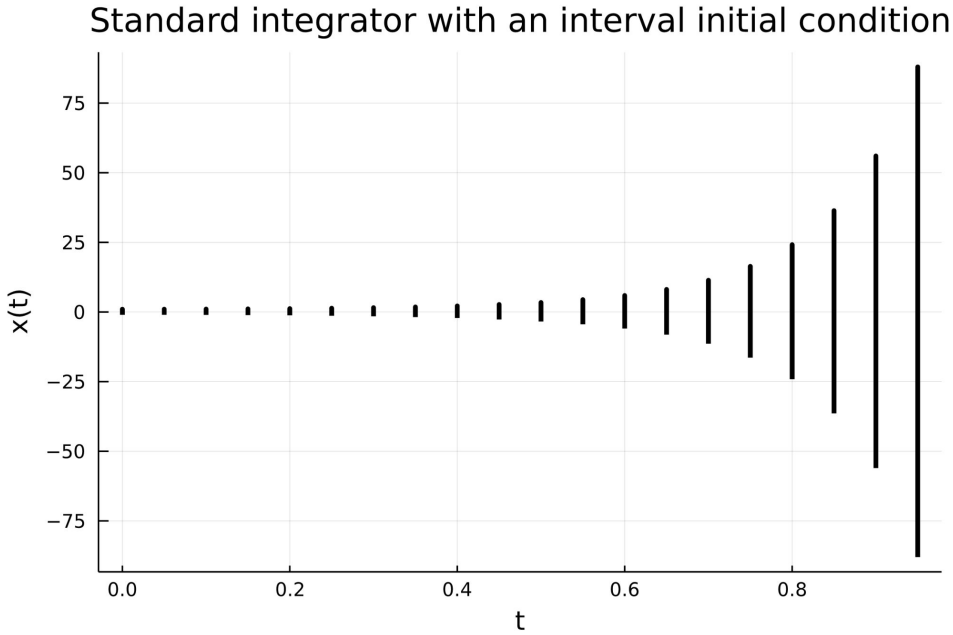
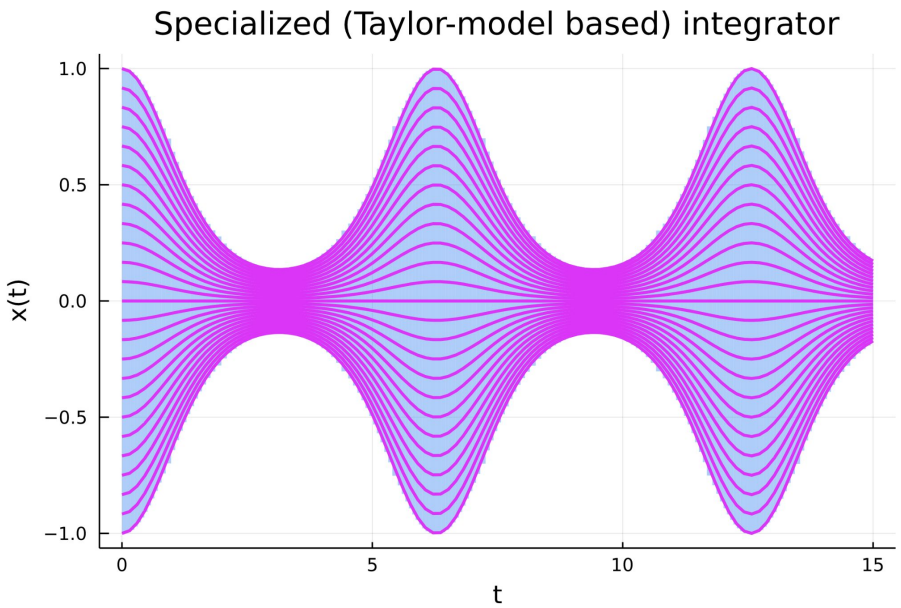
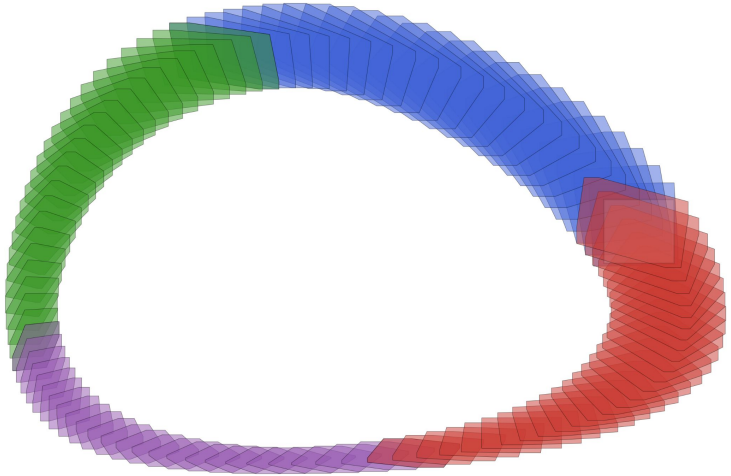
Nonlinear reachability



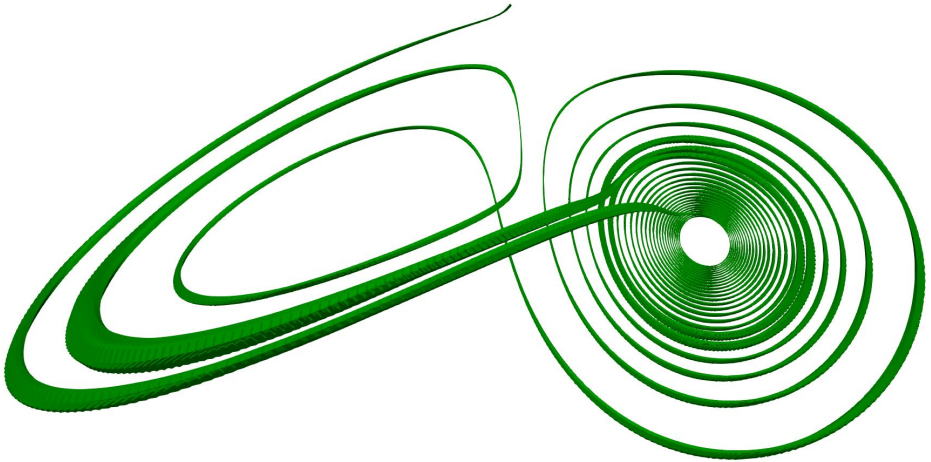


See: *It's all Set: A hands-on introduction to JuliaReach.* Marcelo Forets and Christian Schilling. **JuliaCon'2021 Workshop (available on youtube).**

Nonlinear reachability



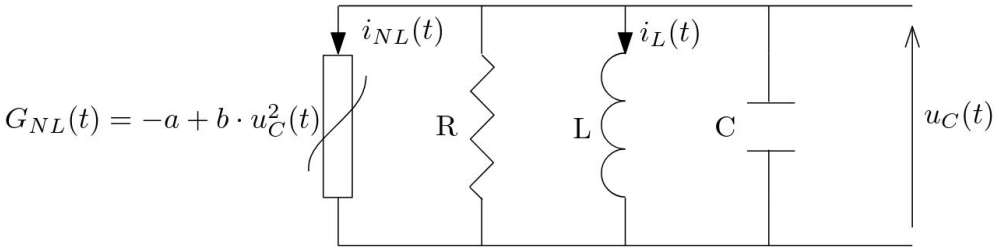
Van der pol system

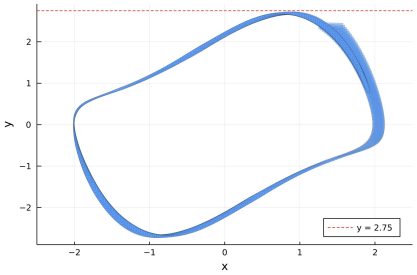


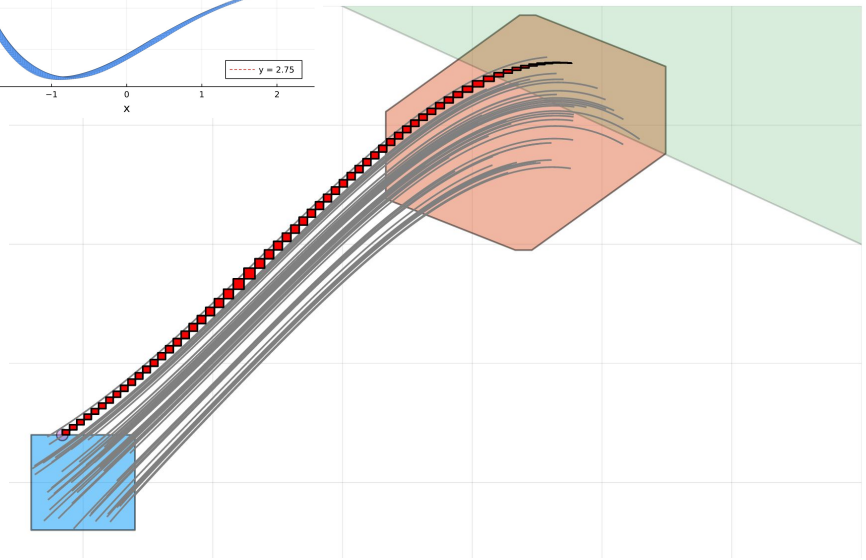
Lorenz system

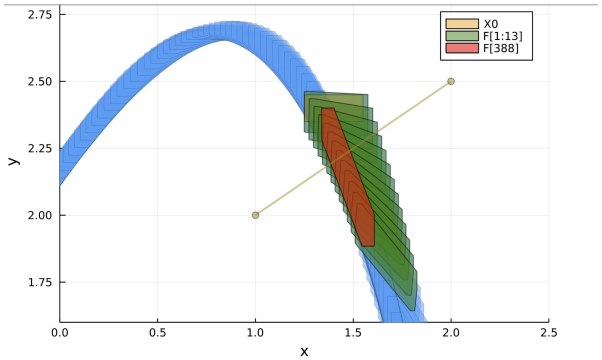
Main nonlinear reachability approaches are: invariant generation, optimization based-approaches, solution-space abstractions, and state-space abstractions.

Parametric reachability









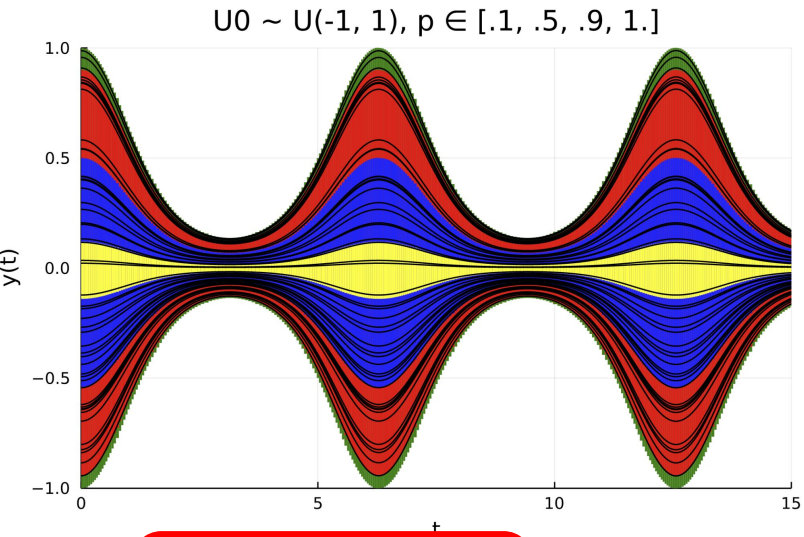
Optimization over reachable states

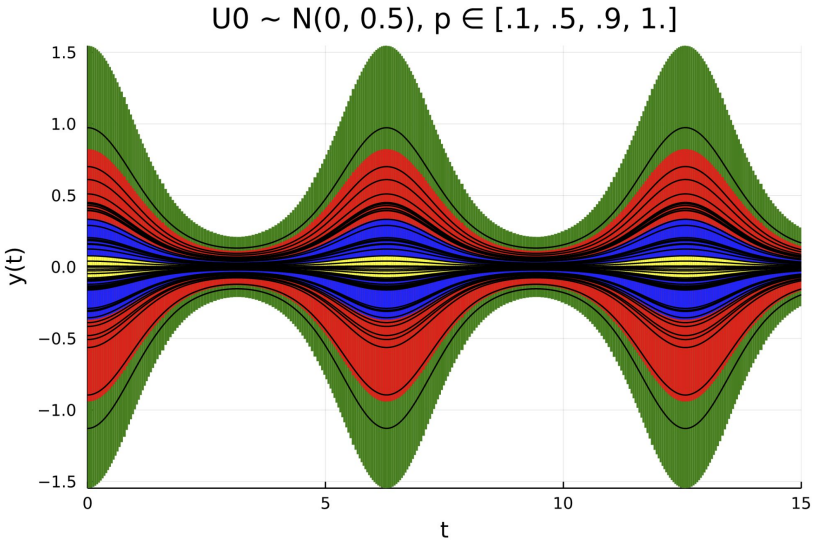
Find falsifying states

Controller synthesis

See: *Parameter sweep of oscillating circuits using Taylor-model flowpipe subset relations (2021).* ***In preparation.***

Probabilistic reachability





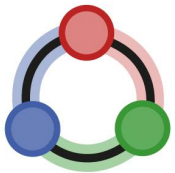
Find enclosing probability bounds

Flowpipe confidence intervals

Propagate initial cdf through a differential equation at time intervals

See: *Verified propagation of imprecise probabilities in nonlinear ODEs (2021).* **In preparation.**

What we aim to do



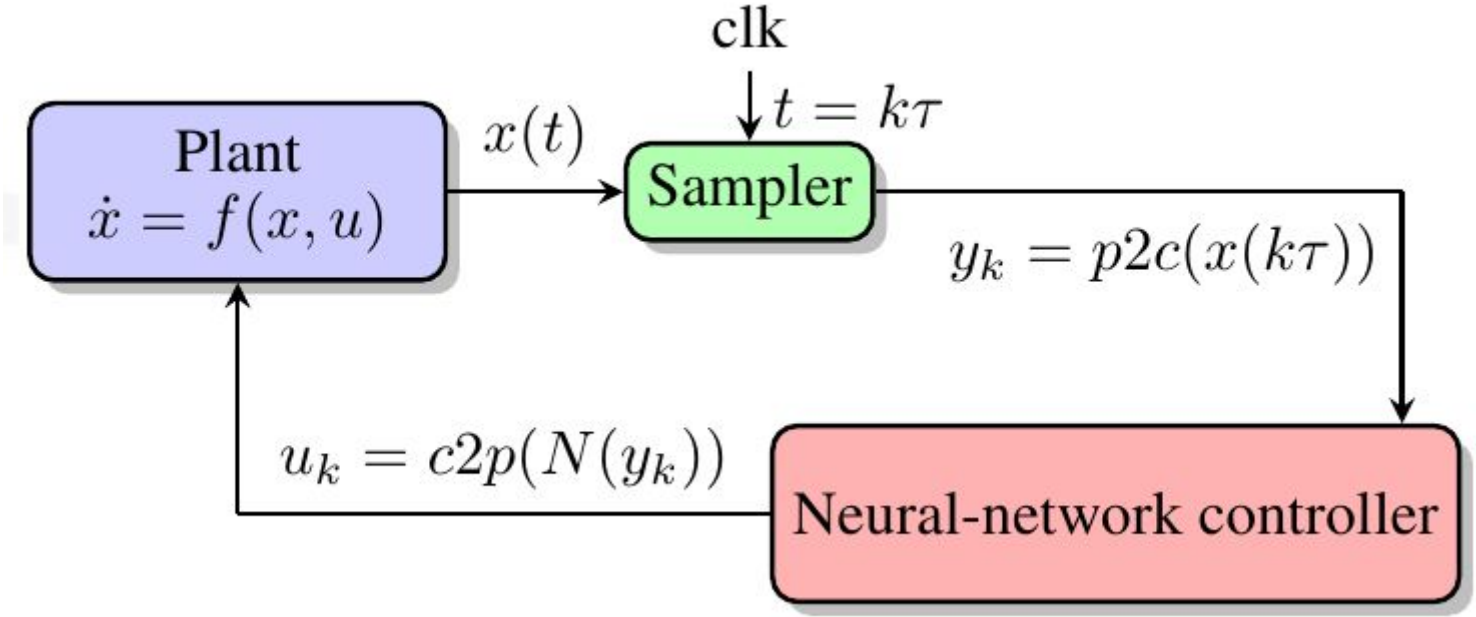
JuliaReach

Advance the state-of-the-art working on *fundamental* problems

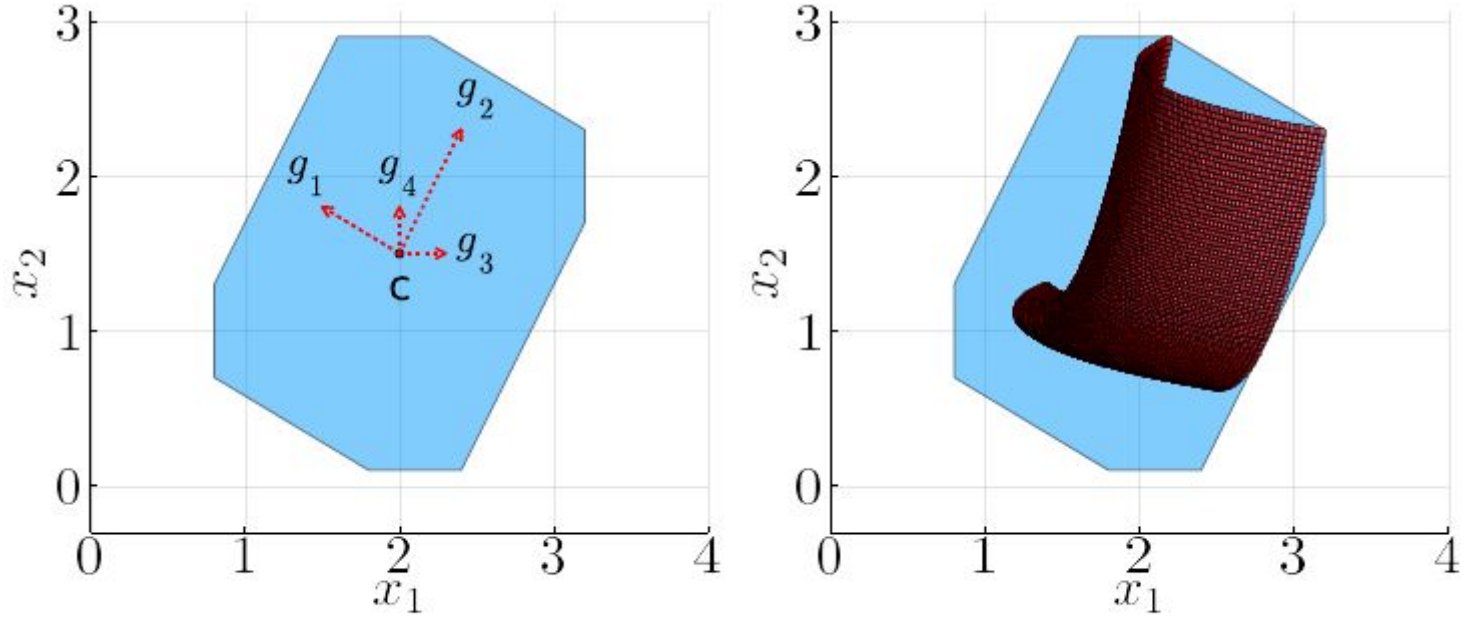
Build comprehensive, efficient, correct, reproducible, well documented libraries

Widen the applicability of reachability analysis for scientists & engineers

Neural-network control systems



Set representations for neural-network control systems



(a) Zonotope.

(b) Taylor model.

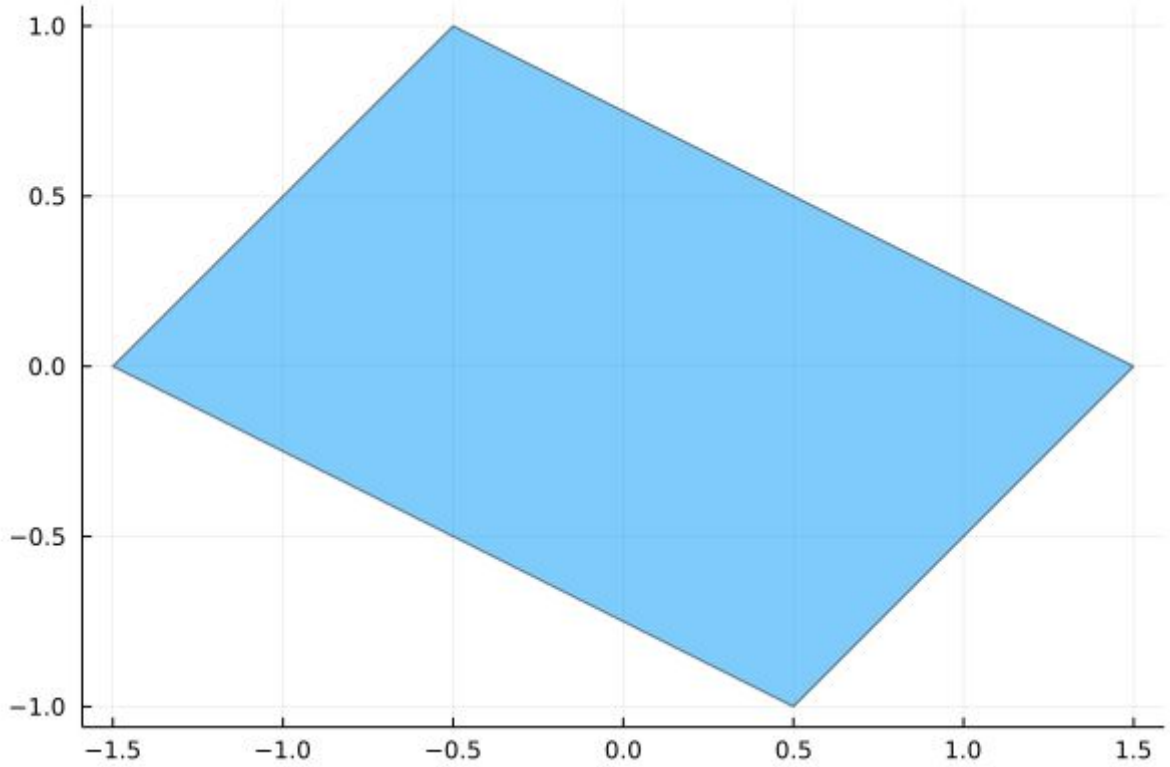
Value propagation in neural networks

$$\sigma(Wx + b)$$

$$\sigma(v_1, \ldots, v_n) = (\sigma(v_1), \ldots, \sigma(v_n))$$

$$\sigma(a) = \max(a, 0)$$ ReLU activation function

Set propagation through ReLU operation



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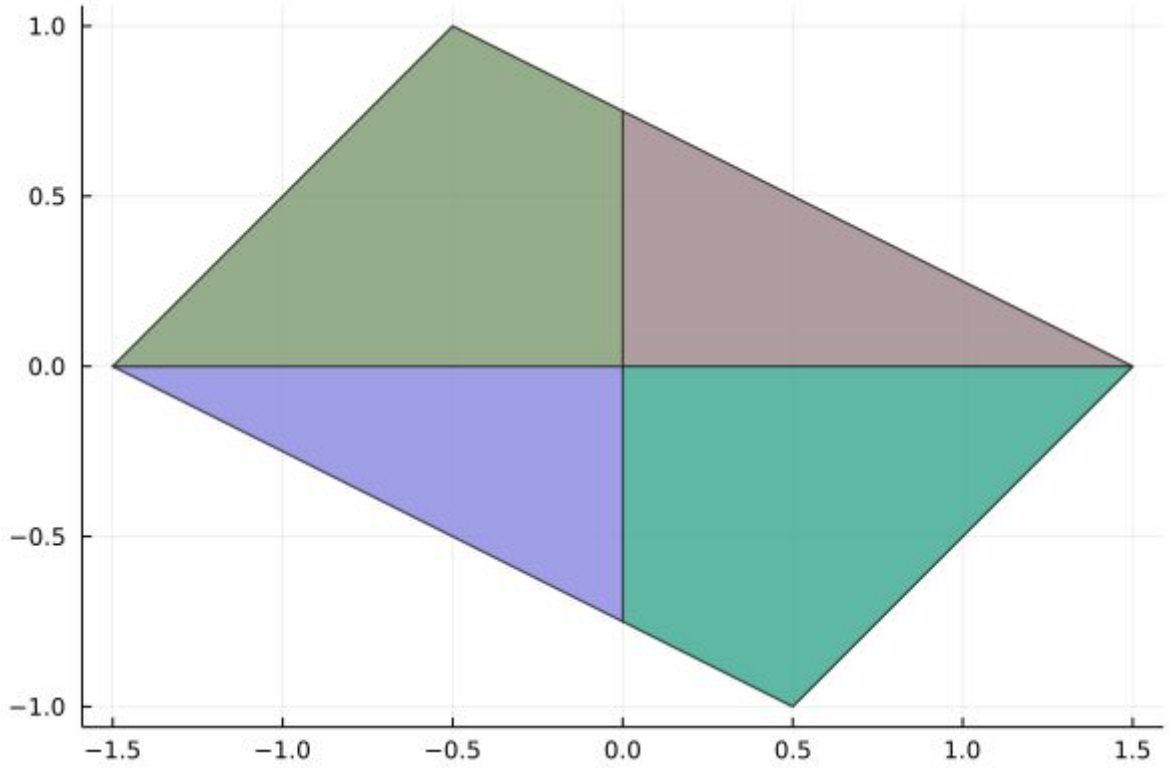
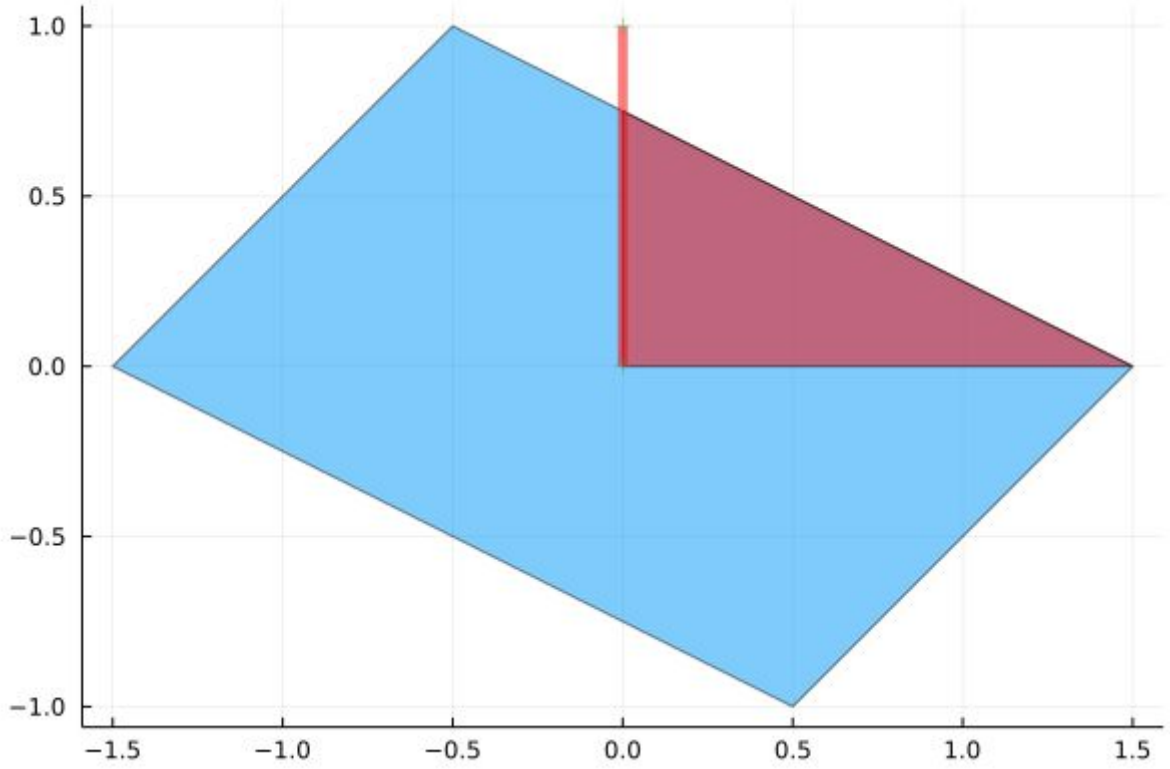


Set propagation through ReLU operation



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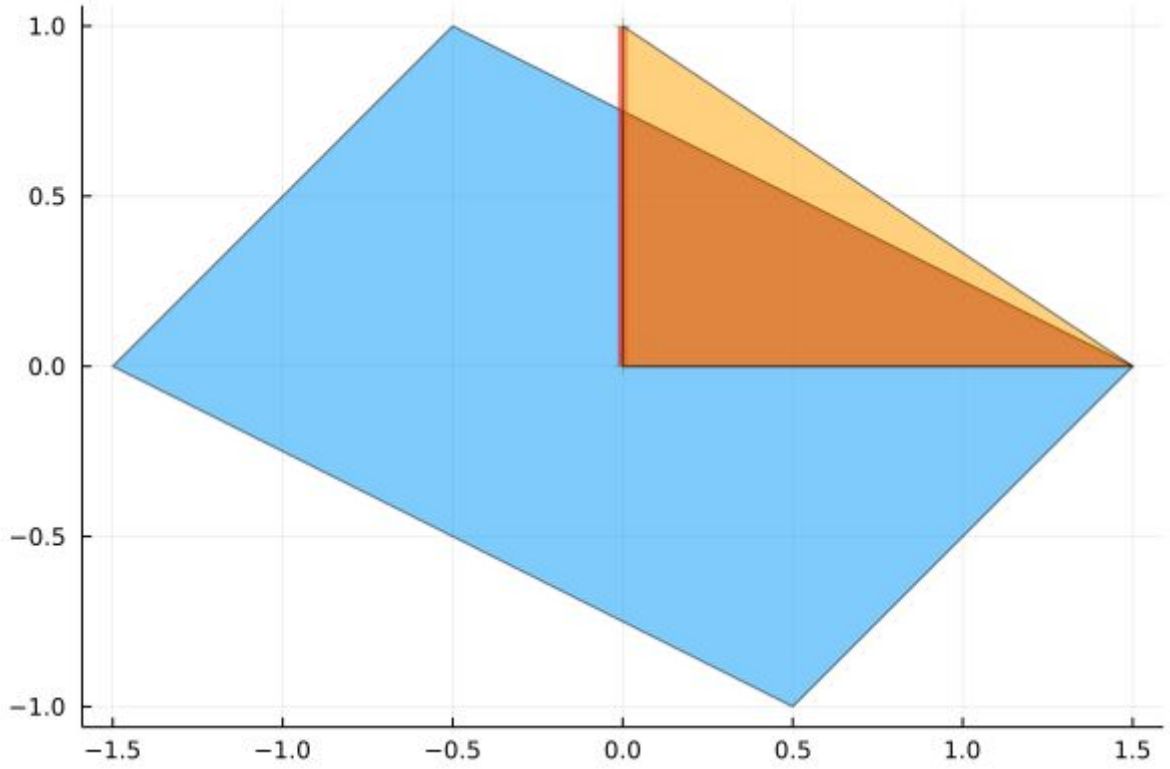
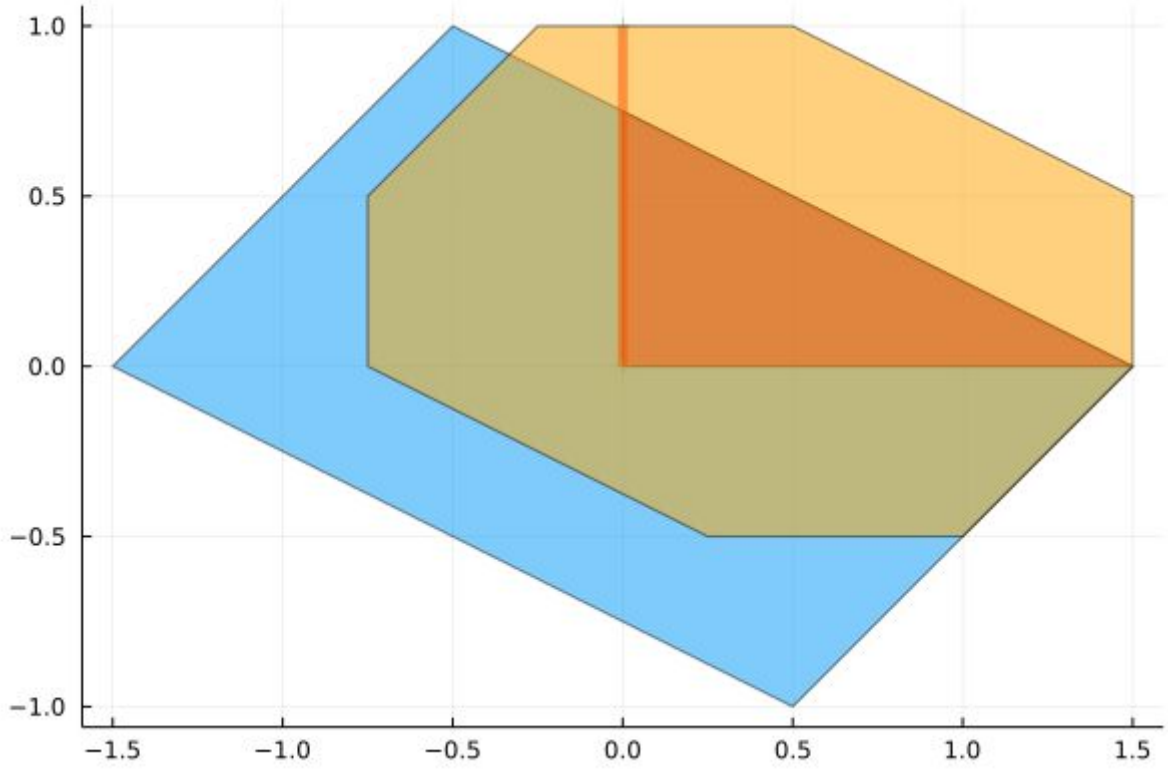
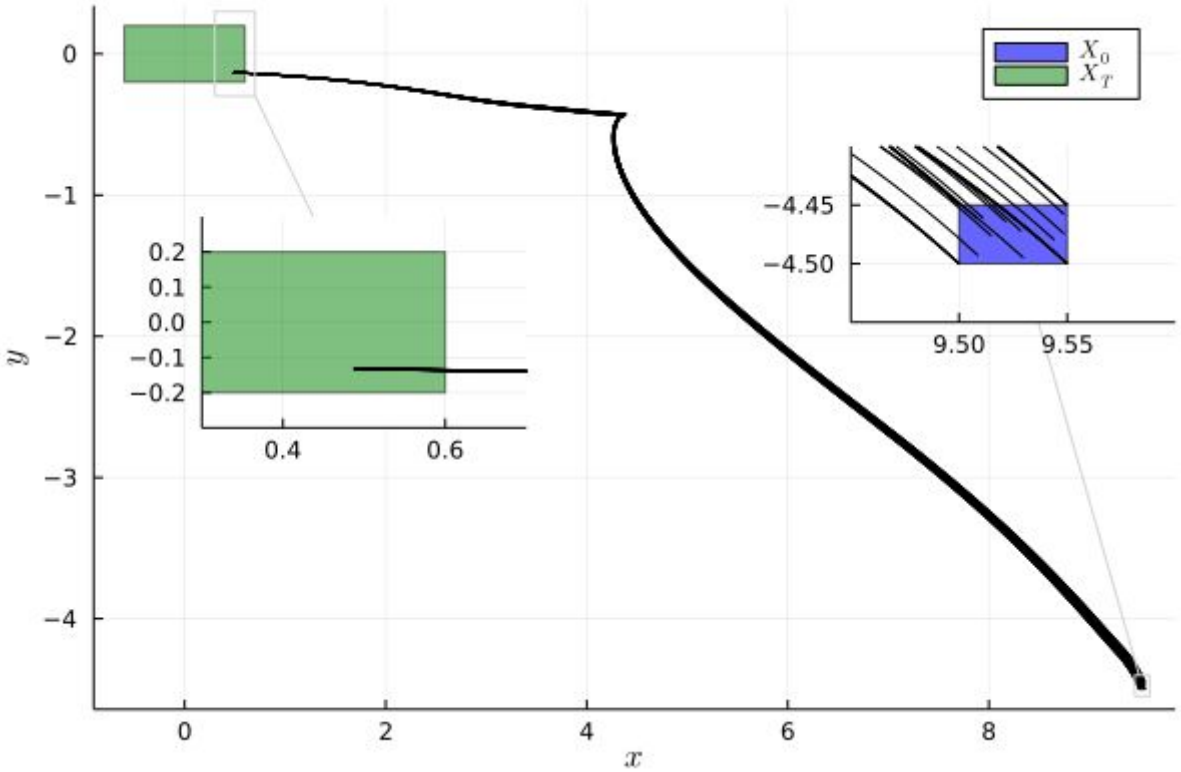


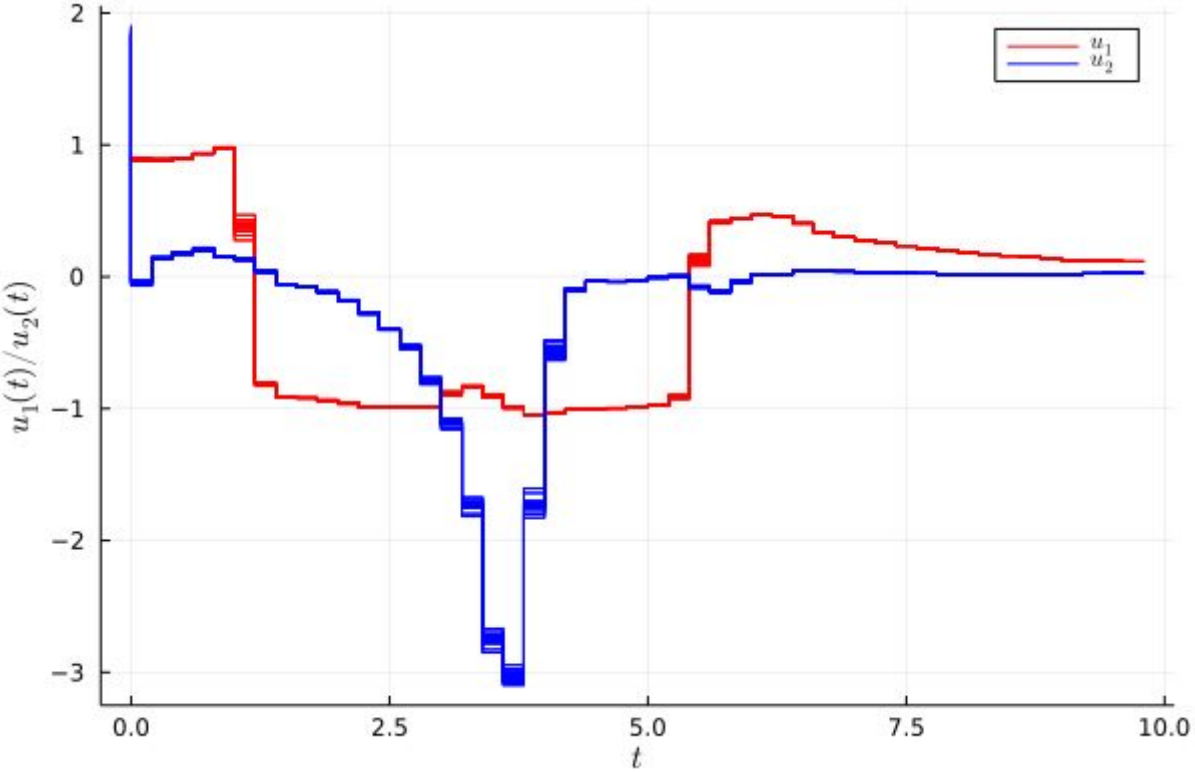
Set propagation through ReLU operation



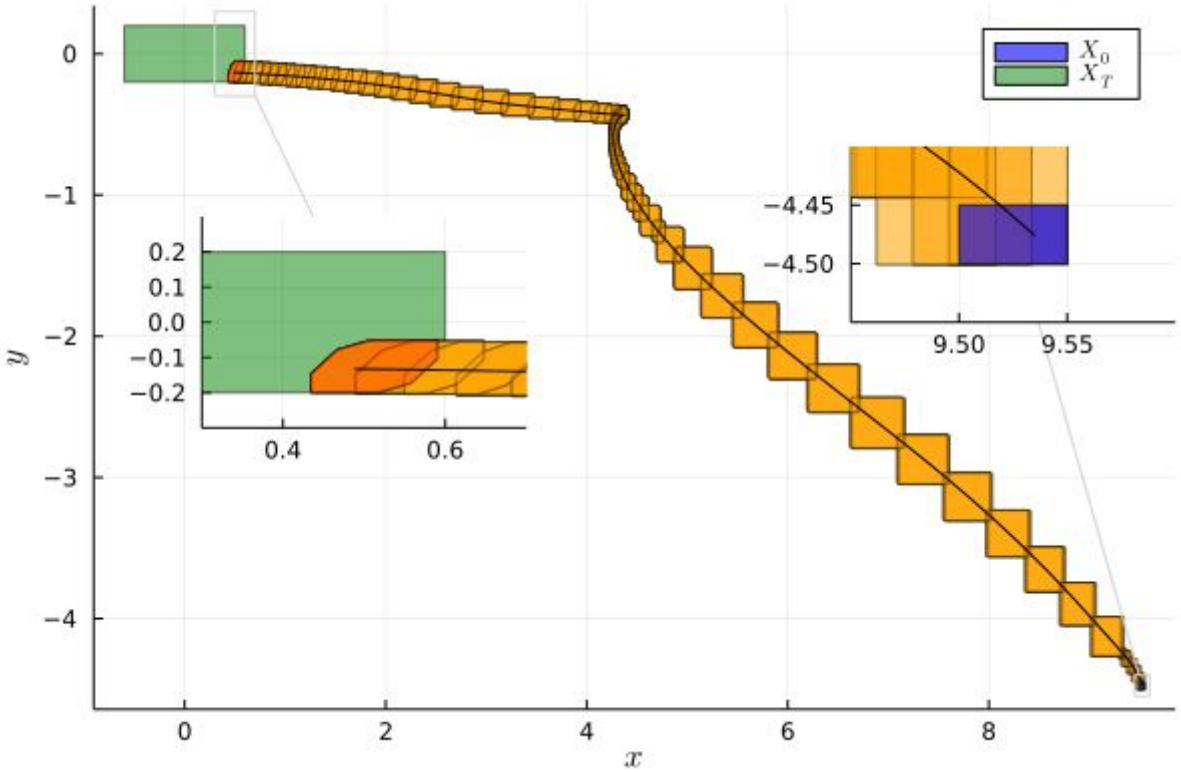
Unicycle model - Simulations



Unicycle model - Control inputs in simulations



Unicycle model - Reachability analysis



JuliaReach

(Verification of neural-network control systems by integrating Taylor models and zonotopes, submitted)